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Taming Triangle Diagrams

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ABSTRACT

A "dialogue" is presented wherein the author demystifies the triangle diagram or trigonometrical graph and shows how it can be a means of discovering hidden relationships between the components of a pyrotechnic composition, finding optimum formulations for a pyrotechnic system, and for summarizing the results of pyrotechnic experiments.

Introduction

Triangle diagrams are not something new; they have been used for well over a hundred years in many areas of scientific study. They can be a powerful tool used to discover relationships between variables that cannot be seen from a tabular listing. Triangle diagrams can help the pyrotechnist quickly find the optimum formulations and can show at a glance the effects of varying formulations. Should the occasion arise to make a written or oral presentation of your studies, you will find triangle diagrams useful in summarizing your work. They can make it easy to demonstrate how you achieved your results. They can also be used to present the results from many experiments in a very compact form.

The current interest in triangle diagrams among pyrotechnists stems mostly from the writing of T. Shimizu (in Lancaster 1972) and Shimizu (1976, 1980, 1982). Shimizu has obviously found triangle diagrams to be extremely useful in both his pyrotechnic research and in his written descriptions of that work. In this article, it is my intention to provide sufficient understanding of triangle diagrams to allow you to read and comprehend the information contained in them. It is further hoped I will convince you that the use of triangle diagrams in your own experimental work will frequently be of great assistance.

This material is written in a style similar to "Programmed Instruction". By that I mean this text is more of a dialogue, between you and me, than would normally be the case. I will accomplish this by asking you to work your way through this text instead of just reading it. I will occasionally ask questions requiring short answers. Occasionally, I have even left blanks for your answers. If you have not experienced programmed instruction before, this may seem a little "hokey", but give it a try. When attempting to learn this type of material (triangle diagrams), programmed instruction will allow you to gain a better understanding in a shorter time. As a check, for your "fill-in-the-blank" answers, I have included answers at the end of the article.

In order to demonstrate that triangle diagrams can make it possible for a pyrotechnist to move faster in developing optimum formulations, consider the hypothetical results listed in Table 1.

Table 1. Hypothetical Results Obtained from Experimental Formulations.

Trial No.	Ingredients by Percent %A	%B	%C	%D	Results
1	45	27	18	10	No effect
2	36	36	18	10	Weak effect
3	27	27	36	10	No effect
4	36	49	5	10	No effect
5	18	45	27	10	Weak effect
6	9	45	36	10	No effect
7	18	54	18	10	No effect
8	27	36	27	10	Weak effect
9	____	____	____	10	Best result

In Table 1, "no effect" means that the formulation did not produce the desired effect or result; "weak effect" means the formulation resulted in some of the desired effects but was less than completely successful. Suppose Table 1 was a listing of the results from your experiments, and you are getting a little pessimistic about the results and a little tired of the project. Suppose you decide to give it one last try; what formulation would you compound? Think about it then fill in the values for trial No. 9 in Table 1. (I won't give you my answer now, because you will have a chance to change your answer later in this article.)

Had you been tracking your experimental progress using triangle diagrams, you could answer the above question almost without hesitation. Also, you would probably pick very nearly that formulation that has the best chance of being successful. In fact, if you had been tracking your experiments on a triangle diagram, you would probably have already tried that "best chance" formulation by trial Number 5 or 6.

Triangle Diagrams

In working up to understanding three component triangle diagrams, consider the more trivial case of working with a two component mixture, having ingredients X and Y. If you wanted to keep track of your progress, you could graph the results using a single line. One end of the line could correspond to 0% X (pure Y) the other end to 100% X (pure X). Then points in between correspond to percentage values from 0% to 100% X (see Figure la). A scale has been added to help locate points along the line. In this case, a coarse scale consisting of 0, 25, 50, 75, and 100% was used. Obviously, any convenient scale could be used. In Part b of this figure, consider Point 1, it corresponds to a formulation containing 25% of ingredient X (and 75% Y). Similarly, Point 2 corresponds to 60% X. In this case, it was necessary for you to mentally interpolate to determine the percentage of Point 2. It was obviously between 50 and 75%. It appears to be about 2/5 of the distance between 50 and 75% and that corresponds to 60%. Now, in my graph I chose (for reasons to become clear later) not to care how far off to the side the points are located. That is to say, both Point 1 and Point 3 correspond to 25% of ingredient X used in the formulation. It does not make any difference that 1 is on the line and 3 is off to the left at some distance. All points lying at the same height above 0% X have the same value.

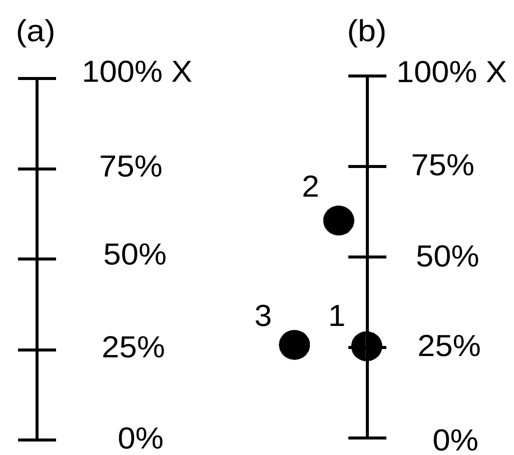

Figure 1. One way of keeping track of experimental results for two component formulations.

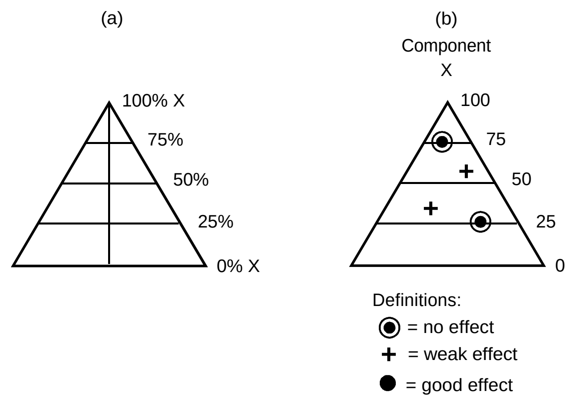

Figure 2. Two component graphs looking a little like triangle diagrams.

In Figure 2a, the scale lines have been widened and a triangular border added, giving the two component graph of Figure 1 more the look of a triangle diagram. Figure 2b has had percent signs and vertical line removed. Again, this was done to achieve the appearance of a triangle diagram. In Figure 2b I have defined three symbols for marking points on the graph. The different symbols indicate varying degrees of success of the two component formulations. There are four points on the graph corresponding to two formulations that exhibited none of the desired effect and two that resulted in a limited success. Starting with the least amount of component X, what are the values, in percent, of X for the four points? (Answers are la, b, c, and d).

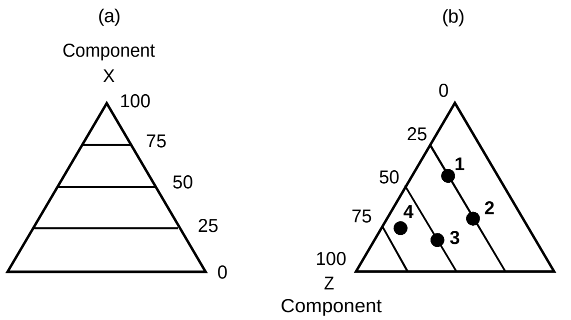

Figure 3. Adding a third component (ingredient) to the mixture and a new scale for measuring it.

Even this very limited information provides a fair idea of the amount of X you should try in your next formulation. The best results so far were achieved with 35% and 60% X; using 25% and 75% X produced no effect. It seems unlikely that trying less than 25% or more than 75% X will produce good results. I would try something in the range from 40% to 50% X. In effect, this would represent splitting the difference between the two formulations that had limited success.

In this two component example, we could have talked about the percentages of ingredient Y just as easily as ingredient X. To do this we would just subtract the percentage of X from 100 to determine the percentage of Y.

In the next example, consider a three component mixture, consisting of ingredients X, Y, and Z. In order to graph mixtures of three components, another scale must be added. This is done in Figures 3 and 4. Figure 3a is simply a two component graph; Figure 3b is the same, but tipped on its side. We read the location of points in Figure 3b for ingredient Z just as we did for ingredient X in Figure 2b. Thus Points 1 and 2 both correspond to 25% of ingredient Z. What are the amounts of ingredient Z for Points 3 and 4? (Answers 2a and b).

Figure 4 is the result of simply overlaying Figures 3a and 3b. Even though this appears considerably more complicated, the only change from 3a and 3b is that 0% and 100% have been dropped from the scales for both ingredients X and Z. This was done partially to avoid confusion in labeling the upper apex of the graph, which is both 100% X and also 0% Z. However, the main reason for this change is because that is the way triangle diagrams most often appear in the literature. You can determine for yourself what the value each apex has by reading that scale increasing toward the apex and mentally adding the next value. It is similar for the scale line forming the side of the triangle opposite each apex. For example (as in Figure 4a), if the scale increasing toward the Z component apex reads 25%, 50%, and 75%, then surely the apex must correspond to 100% Z and the line forming the side opposite the apex must be 0% Z.

Table 2. Three Component Percentages for the Points in Figure 4a.

Point No.	Components		
	%X	%Z	%Y
1	55	25	20
2	35	25	40
3	20	50	30
4	25	65	10

We have already read the percentages of ingredient Z from Figure 3b. It should still be obvious in Figure 4 that the values are 25%, 25%, 50%, and 65% for Points 1 through 4. Using Figure 4a, what are the percentages of ingredient X? (Answers 3a, b, c, and d). If you are having trouble, here are some rules to help you:

1. Find the apex (point) of the triangle diagram labeled for the ingredient you wish to read. (In this case, the upper one labeled X.)

2. Mentally concentrate on the series of parallel

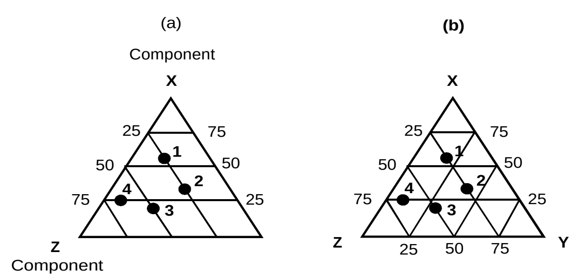

Figure 4. Three component triangle diagram with two and three percentage scales.

lines, starting with the side of the triangle opposite the apex of interest (each line gets shorter as you approach the apex). Refer back to Figure 3a if this gives you trouble.

3. There will be a number indicating a percentage printed at each end of each line. You should use the series of numbers that increase as you approach the apex. (In this case it is the series of numbers on the right, but be careful; the Figure could have just as easily been drawn reversed with the proper scale on the left instead.)

4. Read the percentages for the point of interest from the scale located in Step 3. If necessary, mentally interpolate the distance between two of the parallel lines found in Step 2.

It should now be clear why in Figure 1b, even though Points 1 and 3 both corresponded to 25% X, it was only the distance along the scale from 0% to 100% that was important. That Point 3 was to the left of the scale had to be overlooked. In Figure 4a, both Points 1 and 2 correspond to 25% of ingredient Z, but they indicate different amounts of X.

In a three component mixture it is reasonable to ask for the percentage of all three ingredients. From Figure 4a we have read the percentages of only two. Knowing that percentages must add to 100, the percentages of Y can be calculated. For example, Point 1 has 55% X and 25% Z, thus it must have (100 –55 – 25 = 20) 20% Y. Table 2 is a listing of percentages for the points on Figure 4a; check to be sure you agree.

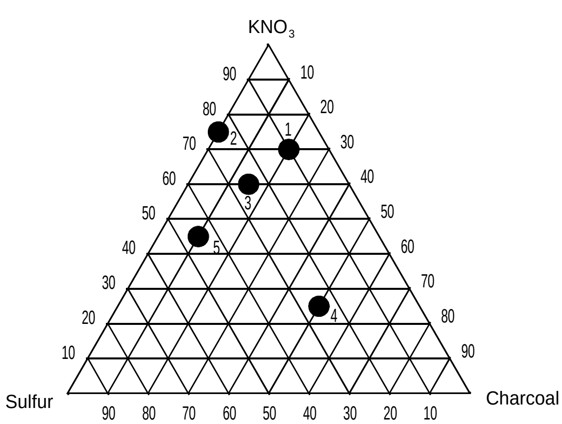

Figure 5. Three component graph for a star prime using black powder ingredients.

Instead of calculating the percentage of the third component, it is normal practice to include an apex and scale lines for each of the three components. This has been done in Figure 4b. Again, the graph was simplified slightly by omitting the word “component”. This is the way you are most likely to see triangle diagrams. Check yourself by reading the Y percentages off the graph and compare with the values in Table 2. If you have trouble, recheck the rules above, this time using the Y apex of the triangle.

As a final exercise before taking up some other aspects of triangle diagrams, consider the triangle diagram in Figure 5. This is a graph for a three component mixture of potassium nitrate, sulfur, and charcoal. In an attempt to confuse you (and simulating the real world) I have reversed (side for side) some of the scales and made them finer. Try filling in the blanks of Table 3. Only through practice will triangle diagrams become understandable and useful. If you follow the rules above, you should not have too much trouble.

Table 3. Tabular Listing of the Data in Figure 5 % of Component.

Point No.	Potassium Nitrate	Charcoal	Sulfur
1	70	20	10
2	75	(4a)___	(4b)___
3	(4c)___	15	(4d)___
4	(4e)___	(4f)___	(4g)___
5	(4h)___	(4i)___	(4j)___

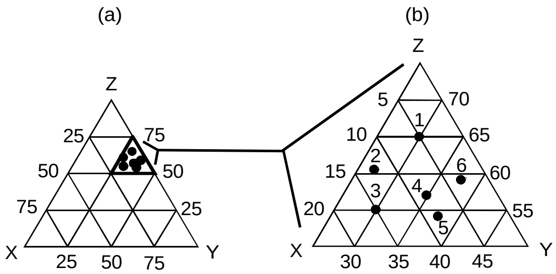

Figure 6. Partial triangle diagrams.

Partial Triangle Diagrams

Consider the data presented in Figure 6a. Here the data are closely grouped in one part of the graph and are difficult to read. If Figure 6a were redrawn as in Figure 6b, expanding the region of interest, it can be more easily read, interpreted or discussed. What has been done is to take the smaller triangle (heavily out-lined in Figure 6a) formed by the lines for 0% X, 25% Y, and 50% Z, enlarge its size and add finer scale lines. The same general rules (listed earlier) still apply. Thus the percentages for Point 1 are 5% X, 30% Y, and 65% Z. Those for Point 2 are 12% X, 27% Y, and 61% Z. If you don't arrive at these same results, recheck the rules. Fill in the missing values for the other points in Table 4.

Table 4. Tabular Listing of the Data in Figure 6.

Point No.	Components %X	%Y	%Z
1	5	30	65
2	12	27	61
3	15	(5a)___	(5b)___
4	(5c)___	(5d)___	57
5	(5e)___	38	(5f)___
6	(5g)___	(5h)___	(5i)___

Formulations with Four or More Ingredients

Using two dimensional triangle diagrams (like we have been using) it is only possible to work with three components at a time. One solution to working with four (or more) ingredients is simply to work with various amounts of the three most critical ingredients while holding the other(s) constant. Good candidates for ingredients to be held constant are the binder or the chlorine donor. Having picked the ingredient to be held constant, guess the proper amount to use (e.g., 5% dextrin). Then always using that amount, experiment to determine the amount of the other ingredients that give the best result. Use a triangle diagram to record the mixtures tried and the degree of success achieved. Next, holding the ratios of these three critical ingredients constant, try using slightly more or less of the ingredient originally held constant. Once the optimum amount has been found, check to see if additional small changes in the other three ingredients are necessary. This last step is particularly important if the optimum amount found for the constant ingredient was much different than the amount used in your first series of experiments.

In practice, there are two slightly different procedures that can be followed when working with four components. You can refigure the percentage of the ingredients so that the three components you wish to vary add up to 100%. Then the fourth component (the one being held constant) can be listed as an "additional percentage" above the 100%. For example, the formulation for a star prime consisting of potassium nitrate, charcoal, sulfur, and dextrin could be given as 75% potassium nitrate, 15% charcoal, and 10% sulfur (equaling 100%) plus an additional +5% dextrin. Additional percentages are usually indicated by the use of a plus sign before the number. In order to work with the data from Table 1, the percentages for the four components could be recalculated, listing ingredient D in terms of an additional percentage. Once this was done the formulations and results could be plotted in a triangle diagram like the three component mixtures shown in Figures 4, 5, and 6 above. This is the technique usually followed by Shimizu. If you are used to working with "additional percentages" and customarily record experimental mixtures this way, then I suggest this is the best way to present your results on a triangle diagram. On the other hand, if you tend to work in parts (and not percent) or if you have the total of all your ingredients equal 100%, then there is a little simpler way to go about it. The three components listed on a triangle diagram can be expressed by parts or percentages that do not add to 100%. However they MUST always add to the SAME number in all formulations to be plotted on the same graph; it's all right if all formulations add to 90%, 80%, 10 parts, or 12 parts, etc. The easiest way to work with the data in Table 1 is to have the components in the triangle diagram always add to 90%, with component D held constant at 10%. Figure 7 is a presentation of the data in this manner.

There is another way to treat mixtures of more than three components on a triangle diagram. This technique uses constant concentration mixtures of ingredients in place of pure components. For example, suppose you wish to use potassium perchlorate and red gum as oxidizer and fuel in a color star formulation. First you could experiment with these two components to find an effective ratio of oxidizer to fuel. Having discovered this, you can prepare a supply of the mixture for further experimenting, now treating the mixture as if it were a single (pure) component. On a triangle diagram you could now use as the three components (1) color agent, (2) chlorine donor, and (3) oxidizer–fuel mixture. Further expanding on this example, suppose that you were attempting to derive a purple star formulation, or one requiring a pair of color agents. You could experiment to find a satisfactory ratio of the color agents, one producing the desired color. Next prepare a mixture of the color agents for future experimenting. Then use both the oxidizer–fuel and the color agent mixtures as single components in your final experimenting. Having found the optimum formulation using mixtures as components, it would be a good idea to check to see if the ratios of ingredients used in the original mixtures could be adjusted slightly to further improve the formulation. This would be particularly important if, for example, the chlorine donor or binder can act as a fuel or if flame temperature affects the color generating ability of the two color agents differently.

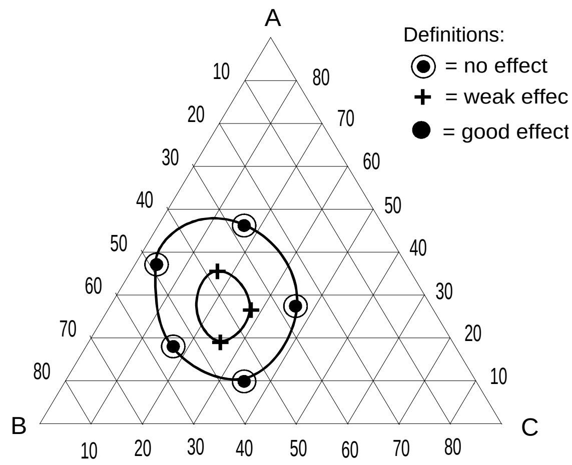

Figure 7. Triangle diagram of the data from Table 1.

Using constant component mixtures of ingredients in triangle diagrams can be a powerful tool. This is because it will allow you to simultaneously work with large numbers of ingredients. In the above example, had you included a binder and a flame deoxidizer as fixed added percentages, you could have been working with seven ingredients on the same triangle diagram.

Conclusion

In Figure 7, a triangle diagram of data from Table 1, I have used different symbols to identify those formulations that produced either no effect or produced a weak effect. I have also added two roughly circular lines, passing through those points giving similar results. It is likely that new formulations with points falling on or near the circles will provide results similar to the other points on the circles although you can not be certain unless you verify it experimentally. In Figure 7, place a point where you would indicate a mixture containing 45% A, 36% B, and 9% C. This point is roughly on the same circle with those that produced none of the desired result. It is rather likely this mixture would also give none of the desired result. What degree of success would you expect from a mixture containing 35% A, 40% B, and 15% C (answer 6)?

It is possible to think of the circles in Figure 7 as a target or "bull's eye". When you look at it in this way, it should be easy to pick your next formulation, that which has a high probability for success. It makes sense to pick one near the center of the bull's eye, or one having concentrations close to (7a) ___% A, (7b) ___% B, and (7c) ___% C. You might choose to compare these percentages with those you selected earlier and listed at the bottom of Table 1. As the result of being clever (or lucky) your earlier guess might almost exactly equal those percentages you just chose. If this is the case, ask yourself these questions:

1. Did you take longer making up your mind studying Table 1 or Figure 7?
2. After looking at Figure 7, are you more confident of the likelihood of having success with your next trial formulation?
3. Depending on the degree of success of this formulation, if still another attempt is necessary, would you pick your next formulation using Table 1 or Figure 7?

If your original "guess" percentages were more than a few percent from your "informed choice" percentages, the value of the triangle diagram should be obvious.

As useful as triangle diagrams can be in guiding your experimentation, they can be even more useful, if you choose to document your investigations. This is equally true whether you are just writing up results for your own future reference, presenting the data orally to an audience, or writing for publication. You can present a large amount of complex data in a single, easily understandable, triangle diagram. Probably you will also wish to include a table of your results, but it is the triangle diagram that will clearly show the complex relationship between the components and the results produced.

Acknowledgments

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Answers

la, b, c, d. (25% X, 35% X, 60% X, and 75% X)

2a, b. (50% Z and 65% Z)

3a, b, c, d. (55% X, 35% X, 25% X, and 20% X)

4a, b, c, d, e. (0% charcoal, 25% sulfur, 60% potassium nitrate, 25% sulfur, 25% potassium nitrate)

4f, g, h, i, j. (50% charcoal, 25% sulfur, 45% potassium nitrate, 10% charcoal, 45% sulfur)

5a, b, c, d, e. (30% Y, 55% Z, 8% X, 35% Y, 8% X)

5f, g, h, i. (54% Z, 3% X, 38% Y, 59% Z)

6. (weak effect or some of the desired result)

7a, b, c. (27% A, 41% B, 22% C)