# Movement Dynamics Of Firework Shells Fired From Mortars

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**Abstract:** *Experimental investigations were carried out using 125 and 310 mm firework shells. The pressures in a mortar during firing were measured. A mathematical model of the firing action concerning low pressure in the space behind the shell was developed. Here it is supposed that non-simultaneous processes of ignition and combustion of black powder particles as well as outflow of combustion materials through the gap between the shell and mortar walls limit the pressure increase. We determined the usage limit of zero-dimensional mathematical models of firing, defined kinetic characteristics for the process of flame spreading through black powder particles, analyzed the results and carried out a comparison between the calculated and experimental results of shots for firework shells of 125 and 310 mm. The calculation results were used to design 2A85 self-propelled mountings for launching shells with integral lifting charge and for the improvement of 2A30 self-propelled mountings for launching shells with integral lifting charge.*

**Keywords:** *Mathematical model, firework, ignition, combustion, flame, powder, muzzle velocity, self-propelled launcher, shell, self-propelled mounting*

## Introduction

In this work we report the experimental results and the simulation of firework movement in firing from a mortar. It is supposed that the pressure is produced in the process of non-simultaneous ignition and combustion of black powder particles and this is followed by outflow of combustion materials through the gap between the shell and the mortar walls. In this case the pressure does not exceed several tens of bars. In firing fireworks from mortars, approaches developed for artillery are usually used. This problem differs from the traditional problems of firing artillery shells. In weapons, where charge density is rather high, the conditions for momentary ignition of powder particles arise. Under the conditions of fast increasing pressure (to about several thousand bar) powder particles burn down the space behind a shell more quickly than the shell comes out from the barrel. Nevertheless, for analysis of the problem under present consideration a traditional statement of the problem of artillery shells is used as a rule with the introduction of some factors that allow fitting theoretical results to the experimental data. To obtain experimentally based results the definition and solution of the problem regarding fireworks fired from a mortar must be considered taking into consideration the basic features of the processes going on during firing. The necessity of its fulfillment is mainly defined by the practical value of results for understanding the ongoing physical processes and for making soundly based decisions for the development of firework shells and mortars for launching them.

## Definition of the problem of artillery firing

Reasonable usage of this or that model is worth evaluating to determine the limits of application of different approaches to solving the problem posed. So, the theoretical study of the artillery firing process is supposed to use a traditional model. One of such models describing the shot process in a thermodynamic approximation is represented.[1] According to this model the kinetic characteristics of the shell can be found from the solution of the following combined equations, including the fundamental equation of pyrodynamics (1), the charge burning law (2), gas formation law (3), and the equation of shell motion (4):

$$PS(l_\psi + x) = fm\psi - \frac{k-1}{2}\varphi MV^2 \qquad (1)$$

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$$\frac{de}{dt} = a_1 P \tag{2}$$

$$\psi = \chi z + \lambda z^2 \tag{3}$$

$$\varphi M \frac{dV}{dt} = PS \tag{4}$$

where $x$ = firework path inside the barrel, $V$ = shell velocity, $P$ = pressure of combustion materials in the space behind the shell, $\frac{de}{dt}$ = speed of black powder combustion, $S$ = barrel cross-section area, $M$ = shell mass, $m$ = lifting charge mass,

$$1_\psi = \frac{1}{S}\left( W - \frac{(1-\psi)m}{\rho} - \alpha m \psi \right)$$

= free reduced length of space behind the shell, $\psi$ = proportion of the powder that becomes gas, $m\psi$ = gas mass, $\alpha$ = covolume, $\rho$ = powder density, $W$ = chamber volume, $f$ = powder force, $k = c_p/c_v$, $c_p$ and $c_v$ = specific heat capacities under constant pressure and volume, $a_1$ = combustion rate of powder under $P$ = 1 bar, $\varphi$ = 1.05 to 1.1 = factor experimentally determined in other work, $z = e/e_0$ = thickness ratio of powder, $e$, $e_0$ = current and initial thickness of powder, $e\chi$, $\lambda$ = factors characterizing the form of powder particles. The adequacy of the proposed model was proved[1] by comparison of the shell velocity on exiting the barrel and the maximum pressure of combustion materials in the barrel with the appropriate values obtained experimentally. Under conditions of low pressure in the space behind the shell in combined equations (1)–(4), as a rule an additional factor $\phi$ is used with the help of which one attempts to take into consideration the non-simultaneity of powder particle ignition and outflow of combustion materials through the gap between the shell and the mortar walls. At that point the factor of outside forces becomes: $\varphi = \phi \times$ (1.05 to 1.1). The value of $\phi$ in each individual case is found from comparison of the experimental data on pressure measurement in the barrel with the calculation results. And if it is possible to achieve satisfactory coincidence of the pressure values with the experimental results in the middle of the process in terms of time, then during the initial moments the calculated values greatly exceed the data obtained in the course of the experiment. The use in practice of such estimated values for firing results particularly in an inevitable weight increase and rise in price of mortars for fireworks, the impossibility of calculating the lifting charge mass for shells, and as a consequence difficulty in achieving the necessary burst height of the shell.

One of the possible reasons for the disagreement between the values obtained in the course of calculation and those determined by experiment lies in the averaging methods that were used in the definition of the problem. The pressures included in equations (1) and (4) are considered to be equal although in equation (1) it is a value that was averaged over the volume of the space behind the shell, and in equation (4) it is a value that is effective at the boundary between the space behind the shell and the volume occupied by the shell. The use of equations (1) and (4) together indirectly implies homogeneity of pressure in the space behind the shell, i.e. some average of medium parameters.

There are detailed discussions[2] on the use of average processes in solving problems of gas dynamics. It is indicated there that "by every average, i.e. by reducing of parameter number characterizing the flow, not all the properties of considerably uneven flow can be retained; some of these properties are lost during average, therefore in some cases average is impossible at all, in other cases the number of parameters describing average flow can differ". Under these constraints the evaluation of pressure inhomogeneity in the space behind the shell is carried out. The gas flow equation (gaseous combustion materials of powder and oxide particles are considered to be so small that they have the speed and temperature of the gas) has the appearance:

$$\rho \frac{dv}{dt} = -\frac{\partial P}{\partial x} \tag{5}$$

Here $\rho$ = gas and particle flow density, $v$ = its speed, $x$ = longitudinal coordinate, $t$ = time. For the velocity of sound the following relation is used:[5] $a^2 = kP/\rho$; from which the density is found and substituted into the equation of motion (5). The equation becomes of the form:

$$\frac{dv}{dt} = -\frac{a^2}{kP} \cdot \frac{\partial P}{\partial x} \quad (6)$$

In order to get the final expression the resulting equation can be written:

$$\frac{P_2 - P_1}{P_0} \approx -k \frac{v_в^2}{a^2} \quad (7)$$

Here $P_0$, $P_1$, $P_2$ = average pressure in the space behind the shell, in the base of the barrel and on the surface of the shell, $v_в$ = average velocity of the gas flow in the cross-section of the barrel near to the shell. For an artillery shell $v_в$ is the movement velocity of the shell.

It follows from the ratio (7) that pressure inhomogeneity in the mortar is proportional to the square of the speed of the shell and inversely proportional to the speed of sound in the combustion materials. The equation (7) actually establishes the limits of applicability of zero-dimensional problems of gas dynamics in mathematical modeling of a shot from a mortar or from an artillery barrel. The essence of these restrictions lies in the finite nature of the rate of information in the volume of gas. A shell that moves through the barrel at supersonic speed is not affected by pressure changes in the base of the barrel.

Since the muzzle velocity of a firework is less than 200 m s$^{-1}$, the error in pressure determination in equation (4) does not exceed 4–6%. Thus the disagreement between experimental data and calculated results is determined by:

1 the non-simultaneous nature of ignition of the powder particles in the space behind the shell;

2 the outflow of powder combustion materials through the gap between the shell and mortar walls.

This article is devoted to the analysis and detailed description of the processes accompanying firework shots under specified conditions.

## Problem of powder combustion polydisperse mode

The investigation relates to mortars that consist of a metal pipe with a welded bottom. The shell with an attached container where the lifting charge in the form of a powder sack is situated is inserted in the mortar (see Figure 1). After actuation of the ignition device, ignition and combustion of some part of the powder particles on reaching destruction pressure inside the container, the combustion action spreads in the space under the shell. As a result of the pressure from the combustion of the lifting charge, the firework moves along the mortar barrel till the moment it emerges into the atmosphere. After leaving the barrel the shell does not accelerate any further.

The area where the parameters of the problem are determined is the volume (Figure 1) bounded by the inner surface of the barrel, the lower surface of the firework shell and the cross section of the barrel axis that has minimum clearance between the firework shell and the barrel (critical section). One or two electric matches activate the black powder in the lifting charge. At the initial moment only some part of the powder charge ignites. With time the flame spreads along the whole powder charge involving more and more powder particles

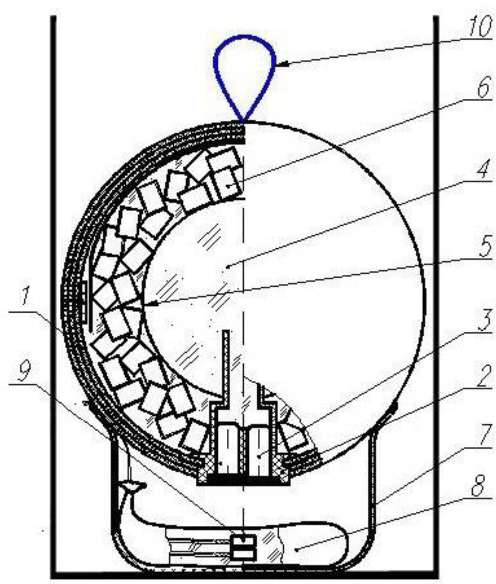


**Figure 1.** *1 – Firework shell; 2 – Pipe; 3 – Delay mechanism; 4 – Ignition-bursting charge (IBC); 5 – Casing of IBC; 6 – Pyroelements; 7 – Lifting charge container; 8 – Lifting charge (LC); 9 – Electric match; 10 – Loop.*

in combustion. Gaseous and condensed products of powder combustion are formed. The mass fraction of condensed material is quite substantial – 0.56 – and it must be considered in the process of analysis. It is suggested that the particles in the condensed phase are of a size at which it is possible to assume that both gas and condensed phases have the same temperature and velocity. Under the pressure influence of the gaseous products of powder combustion the firework moves and is ejected into the atmosphere. At that point a proportion of the combustion products emerge into the atmosphere through the gap between the firework shell and the inner surface of the barrel, reducing the pressure level in the space behind the shell. The parameters of the suggested model are defined from the solution of the following combined equations consisting of the mass change equation (8), energy equation (9), equation of firework motion (10) and equation of state (11):

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \dot{\psi} - G \tag{8}$$

$$\frac{\mathrm{d}(mu)}{\mathrm{d}t} = -P\frac{\mathrm{d}v}{\mathrm{d}t} + \dot{\psi}\, u_0 - Gu \tag{9}$$

$$M\frac{\mathrm{d}V}{\mathrm{d}t} = PS_{\mathrm{u}} \tag{10}$$

$$P = (1 - \varepsilon)(m/v)(Ru/c_{\mathrm{vg}}) \tag{11}$$

Here $m$ = mass of powder combustion products, $\varepsilon$ = fraction of powder combustion products in the condensed phase, $\dot{\psi}$ = mass input of powder combustion products per unit of time, $G$ = mass flow of powder combustion products through the surface of critical section, $u$ = internal energy per mass unit of combustion products, $u_0 = c_{\mathrm{v}}T_{\mathrm{comb}}$; $c_{\mathrm{v}}$ = specific heat of combustion products under constant volume, $T_{\mathrm{comb}}$ = combustion temperature of powder, $v$ = the volume occupied by powder combustion products, defined by the relation: $v = W - m_{\mathrm{p}}/\rho_{\mathrm{p}} + Sx$; $W$ = volume under the shell at initial time, $m_p$ – mass of unburnt powder, $\rho_{\mathrm{p}}$ = powder density, $S$ = cross-section area of the barrel, $x$ = firework path inside the barrel, $S_{\mathrm{u}}$ = cross-section area of firework shell, $P$ = pressure in the space behind the shell, $R$ = gas constant in gas phase of powder combustion products, $c_{\mathrm{vg}}$ = specific heat in the gas phase of combustion products at constant volume.

In order to define the function $\dot{\psi}$ a number of assumptions were made. It is supposed that powder particles are of spherical form, with a particle diameter of $2r$ = 1 mm. After actuation of ignition initiators some part $N_0$ of the powder particles is ignited. From the burning powder the flame spreads to the rest of the particles. It is supposed that the burning and unburnt powder particles are evenly distributed within the space behind the shell and the volume occupied by the particles is relatively small in comparison with the volume where powder combustion occurs. The mechanism of flame spreading through the particles is represented in the form:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \alpha_1 \cdot N \cdot (P/P_0)^{\alpha_2} \tag{12}$$

*i.e.* the number of inflammable particles per unit volume is proportional to the number of burning particles per unit volume and pressure of combustion products. Here $\alpha_1$ and $\alpha_2$ are constants determined in the course of the experiment. At the initial moment all the particles are of the same size. The mixture is monodisperse. As the flame spreads some particles are ignited, others cease burning. Therefore for the treatment of powder particle combustion it is necessary to use the polydisperse medium model. Equation (12) determines the mechanism of formation of particles of burning powder. Here the function $\delta_i$ is introduced for different fractions of powder. This function is equal to one if the fraction is burning and equal to zero if the burning has not yet begun or is already finished. The number of fraction $i$ of particles being ignited is found by integration of equation (12) with respect to time $[t_i, t_{i+1}]$:

$$N_i = \alpha_1 \int_{t_i}^{t_{i+1}} \left(\sum_{j=0}^{i-1} (\delta_j \cdot N_j)\right)(P/P_0)^{\alpha_2}\,\mathrm{d}t \tag{13}$$

Here $i = 1\ldots J_k$, $t_i = i\Delta$, $t_k = J_k\Delta$, $t_k$ is the time at the end of the process. Equation (13) is true if $\sum_{j=0}^{i-1} N_j$ is less or equal to particle number, otherwise $N_i = 0$. Following Nigmatulin[3] we consider that unitary fuels to which powder and explosive substances refer contain within them not only "fuel" in particular but also an oxidizing agent "mixed" with the fuel on a molecular level; so they represent a condensed solid homogeneous mixture of "fuel" and oxidizing agent. The linear combustion velocity of powder and other types of unitary fuel depends on the pressure. The corresponding empirical dependence has the form as stated by Zeldovich:[4]

$$\frac{\mathrm{d}r_i}{\mathrm{d}t} = -b_1 (P / P_0)^{b_2} \qquad (14)$$

where $b_1$ and $b_2$ are empirical constants, individual for each type of fuel.

For the powder involved: $b_1$ = 12.1 mm s$^{-1}$, and $b_2$ = 0.24, under $P$ < 60 MPa.

At particle combustion of fraction $i$ the value of $r_i$ becomes zero. At the same instant the function $\delta_i$ becomes zero as well. For the function $\dot{\psi}$ the following expression can be written:

$$\dot{\psi} = 4\pi\rho \cdot \sum_i \delta_i \cdot N_i \cdot r_i^2 \cdot \left| \frac{\mathrm{d}r_i}{\mathrm{d}t} \right| \qquad (15)$$

where the sum is carried out on all the fractions.

The function $G$ is found according to Abramovich[5] the solution about combustion materials outflow from supersonic nozzle. A similar problem is examined by Weinman.[6] A supersonic nozzle must consist of convergent (subsonic) and divergent (supersonic) parts. In the narrowest section of a supersonic nozzle (critical section) the flow velocity is equal to the sonic velocity in the combustion materials. The flow of gas or gas mixture through the critical section is determined from the relation:

$$G = \frac{PF_{кр}}{\sqrt{T}} \left( \frac{2}{k+1} \right)^{\frac{k+1}{2(k-1)}} \left( \frac{k}{R} \right)^{0.5} \qquad (16)$$

Here $F_K$ is the critical section between firework product and barrel wall, $P$, $T$ = pressure and temperature of the gas in the space outside the shell, $R$ is the gas constant. The constant for the mixture of powder combustion products is $k \approx 1.1$. From Zeldovich[4] it follows that supersonic flow of powder combustion products begins at $P > 1.63\ P_0$. In practice under the conditions of the shot this value is achieved immediately.

Combining equations (8)–(16) allows the solution of the problem of a shell fired from a gun barrel taking into consideration the non-simultaneity of powder particle ignition and combustion, and the outflow of powder combustion products through the gap between the shell and the inner wall of the gun during the shot.

## Practical use of the solution method developed

The solution of this problem was used for the study of firework shells fired from a mortar barrel and for the design of shells. High-altitude firework shells manufactured in the Russian Federation are subdivided into two groups. One group includes shells of caliber 195 and 310 mm operating at a height of 250–500 m and capable of creating large figures in the sky from several types and sizes of pyro elements. The other group includes shells of caliber 60 mm and 105 mm with operational heights up to 150 m, and these may be equipped with only one type of pyro element because of the size of the shells. In order to make firework displays more attractive it is necessary to have shells that can operate at a height of 150–250 m and form very large figures. The public corporation "Piro-Ross" has developed a firework shell that can achieve this sort of result. The relevance of the development of shells of caliber 125 mm is also determined by the fact that mortars of this caliber can be used for firing existing shells from self-propelled mountings 2A30 for launching shells with integral lifting charge without any change in the main structural features of the shells. Before shell development began, calculation research was carried out. For that the constants including

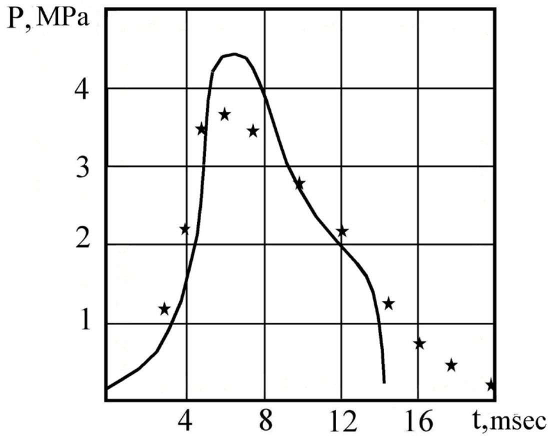


**Figure 2.** *Pressure measured in the lower part of the mortar for a 310 mm shell. Asterisks indicate experimental results and the solid line indicates calculated values.*

in equation (12) were found at the initial stage. The product with a caliber of 310 and 125 was experimentally more thoroughly examined. Experimental results were used to determine the constants $N_0$, $\alpha_1$ and $\alpha_2$ which were determined by the gradient method from the solution created with the help of the model equations (8) – (16).

Figure 2 shows the results of measuring combustion product pressure in the bottom part of the barrel for a 310 mm firework. Asterisks show the experimental results. The solution showed that ~68 g powder out of the total mass of 700 g is ignited by the electric match at the initial moment. The values $\alpha_1$ and $\alpha_2$ are respectively equal to 22 and 1.95. The calculation for a firework of 125 mm caliber presented in Figure 3 shows that the kinetic constants retain their values whereas the mass of powder being ignited at the initial moment depends on the caliber of the shell. The mass is equal to 35 g out of the total mass 70 g for a 125 mm firework. The kinetics of ignition and combustion of powder particles represent those during the firing process for a 310 mm firework ~90% of the lifting charge is burnt off. The rest of the burning powder particles and combustion products fly out into the atmosphere from the mortar barrel after the firework shell and create a burning smoky cloud.

## Conclusion

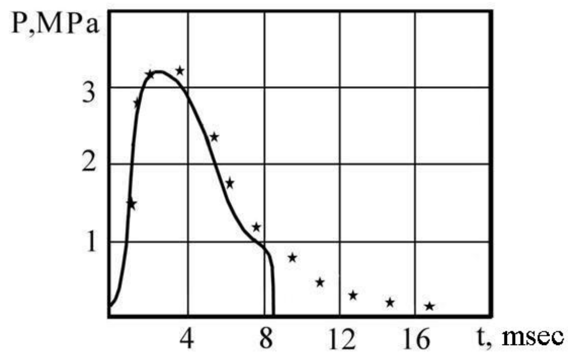


**Figure 3.** *Pressure measured in the lower part of the mortar for a 125 mm shell. Asterisks indicate experimental results and the solid line indicates calculated values.*

We developed a model for firing of fireworks from mortar barrels under conditions of low pressure in the space outside the shell which originates from the non-simultaneity of powder particle ignition and combustion and the outflow of powder combustion products through the clearance between the firework and the inner wall of the mortar barrel. Comparison of the results of the calculation with the experimental data showed that the model could be applied to a wide variety of firework shells used in civilian pyrotechnics. In particular the calculation results from the research allow the conclusion that for functioning of a 125 mm firework at a height of 150–250 m it is enough to use a lifting charge with 72 g powder and a mortar of barrel length 450 mm in the firework setup. The experimental results demonstrated the adequacy of mass of powder used for the lifting charge of the 125 mm firework and the parameters of the launcher. The use of this product will offer the chance to create more colorful and varied firework pictures in the night sky using improved 2A30 self-propelled mountings and new 2A85 self-propelled mountings without increasing the danger areas or changing the foundations of 2A30 self-propelled mountings.

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