Atmospheric Flight Dynamic Simulation Modeling Of Spin-Stabilized Projectiles And Small Bullets With Constant Aerodynamic Coefficients

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Abstract. *A full six degrees of freedom (6-DOF) simulation flight dynamics model is applied for the accurate prediction of short and long range trajectories of high and low spin-stabilized projectiles and small bullets via atmospheric flight to final impact point. The projectile is assumed to be both rigid (non-flexible) and rotationally symmetric about its spin axis launched at low and high pitch angles. The projectile maneuvering motion depends on the most significant forces and moments, in addition to wind, gravity and Magnus effects. The computational flight analysis is based on appropriate constant mean values of the aerodynamic projectile coefficients taken from an official tabulated database. The newer ICAO atmospheric model simulates the height distributions of density, pressure and temperature properties of the air. Static stability, also called gyroscopic stability, is examined as a necessary condition for stable flight motion in order to determine the sufficient initial spinning projectile rotation. The efficiency of the method developed gives satisfactory results compared with published data of verified experiments and computational codes on atmospheric dynamics model flight analysis.*

Keywords: *Trajectory dynamics simulation, constant aerodynamic analysis, high and low pitch angles, Magnus effects, symmetric projectiles, static stability criteria*

Introduction

Ballistics is the science that deals with the motion of projectiles. The word ballistics was derived from the Latin 'ballista', which was an ancient machine designed to hurl a javelin. The modern science of exterior ballistics[1] has evolved as a specialized branch of the dynamics of rigid bodies, moving under the influence of gravitational and aerodynamic forces and moments.

Pioneering English ballisticians Fowler, Gallop, Lock, and Richmond[2] constructed the first rigid six degrees of freedom projectile exterior ballistics model. Various authors have extended this projectile model for lateral force impulses,[3,4] dual-spin projectiles,[5,6] etc.

The present work addresses a full six degrees of freedom (6-DOF) projectile flight dynamics analysis for the accurate prediction of short and long trajectories of high spin-stabilized projectiles and bullets. The proposed flight model takes into consideration the influence of the most significant forces and moments, based on appropriate constant mean values of the aerodynamic coefficients, in addition to wind and Magnus effects. The efficiency of the computational method developed gives satisfactory results compared with published data of verified experiments and computational codes on projectile trajectory analysis with various initial flight conditions at the firing site.

Projectile model

The present analysis considers two different types of representative flight projectile vehicles. A typical formation of the cartridge 105 mm HE M1 projectile is presented in Figure 1 and is used with various 105 mm Howitzers such as M103 with M108 cannon, M137 with M102 cannon as well as NATO L14 MOD56 L5. Cartridge 105 mm HE M1 is of semi-fixed type ammunition, using adjustable propelling charges in order to achieve desirable ranges. The projectile producing both fragmentation and blast effects can be used against personnel and materials targets.

Figure 1. *105 mm HE M1 high explosive projectile artillery ammunition for howitzers.*

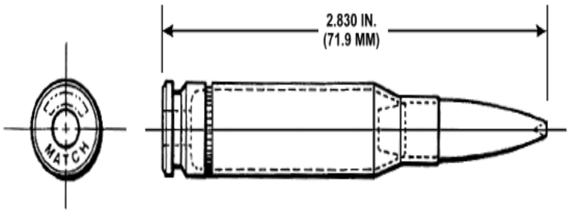

Figure 2. *7.62 mm M852 match ammunition with a diameter of .30 caliber representative bullet.*

Also a 0.30 caliber (0.308 inch diameter), 168 grain (≥10.9 g) Sierra International bullet used in a National Match M14 rifle is loaded into 7.62 mm M852 match ammunition for high power rifle competition shooting, as shown in Figure 2. The cartridge is intended and specifically prepared for use in those weapons designed as competitive rifles and also for marksmanship training. This bullet is not for combat use. The cartridge is identified by the cartridge case head stamping of MATCH. It also has a knurl at the base of the cartridge case and a hollow point boat-tail bullet.

Table 1. *Physical and geometrical data of 105 mm big projectile and 7.62 mm small bullet.*

Characteristics	105 mm HE M1 projectile	7.62 mm bullet
Reference diameter, mm	104.8	7.62
Total length/mm	494.7	71.88
Weight/kg	15.00	0.385
Axial moment of inertia/kg m^{-2}	$2.326 \cdot 10^{-2}$	$7.2282 \cdot 10^{-8}$
Transverse moment of inertia/kg m^{-2}	$2.3118 \cdot 10^{-1}$	$5.3787 \cdot 10^{-7}$
Center of gravity from the base/mm	183.4	12.03

Physical and geometric characteristics data of the above mentioned 105 mm HE M1 projectile and 7.62 mm bullet are illustrated in Table 1.

Trajectory flight simulation model

A six degrees of freedom rigid-projectile and bullet model[7–10] has three rotations and three translations. The three translation components (x, y, z) describing the position of the projectile's center of mass and the three Euler angles (ϕ, θ, ψ) describing the orientation of the projectile body with respect to Figure 3.

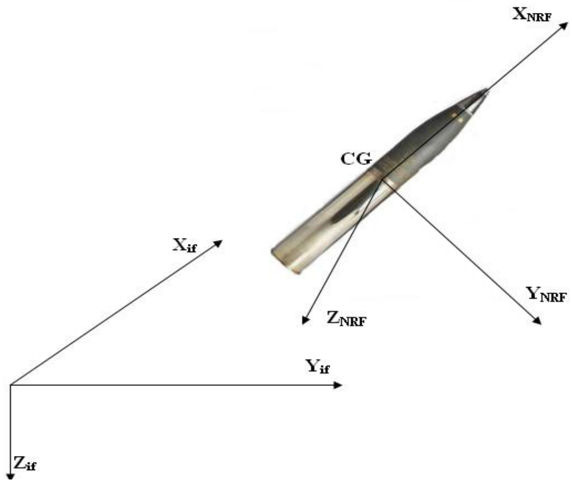

Figure 3. *No-roll (moving) and fixed (inertial) coordinate systems for the projectile trajectory analysis.*

Two mean coordinate systems are used for the computational approach of an atmospheric flight motion. The one is a plane fixed (inertial frame, if) at the firing site. The other is a no-roll rotating coordinate system on the projectile body (no-roll-frame, NRF, $\phi = 0$) with the X_{NRF} axis along the projectile axis of symmetry and Y_{NRF}, Z_{NRF} axes oriented so as to complete a right-hand orthogonal system.

Newton's laws of the motion state that rate of change of linear momentum must equal the sum of all the externally applied forces and the rate of change of angular momentum must equal the sum of all the externally applied moments, respectively. The force acting on the projectile comprises the weight, the aerodynamic force and the Magnus force.

The moment acting on the projectile comprises the moment due to the standard aerodynamic force, the Magnus aerodynamic moment and the unsteady aerodynamic moment.

Therefore, the twelve state variables x, y, z, ϕ, θ, ψ, u, v, w, p, q and r are necessary to describe the position, flight direction and velocity at every point of the projectile's atmospheric flight trajectory. Introducing the components of the acting forces and moments with regard to the no-roll-frame (~) rotating coordinate system with the dimensionless arc length as an independent variable, we derive the following equations of motion for six-dimensional flight:

$$x' = D\cos\psi\cos\theta - \frac{D}{V}\sin\psi\,\tilde{v} + \tilde{w}\cos\psi\sin\theta\,\frac{D}{V} \tag{1}$$

$$y' = D\cos\theta\sin\psi + \tilde{v}\cos\psi\,\frac{D}{V} + \tilde{w}\sin\theta\sin\psi\,\frac{D}{V} \tag{2}$$

$$z' = -D\sin\theta + \frac{D}{V}\,\tilde{w}\cos\theta \tag{3}$$

$$\varphi' = \frac{D}{V}\,\tilde{p} + \frac{D}{V}\tan|\theta|\,\tilde{r} \tag{4}$$

$$\theta' = \frac{D}{V}\,\tilde{q} \tag{5}$$

$$\psi' = \frac{D}{V\cos\theta}\,\tilde{r} \tag{6}$$

$$\tilde{u}' = -\frac{D}{V}g\sin\theta - \frac{\pi}{8m}\rho V D^3 C_X - D^3\frac{\pi}{8m}\rho V C_X{}^2\alpha^2 - D^3\frac{\pi}{8m}\rho V C_X{}^2\beta^2 + \tilde{v}\frac{D}{V}\tilde{r} - \tilde{q}\frac{D}{V}\tilde{w} \tag{7}$$

$$\tilde{v}' = -D^3\frac{\pi}{8m}\rho\,C_{NA}\left(\tilde{v}-\tilde{v}_W\right) + D^4\frac{\pi}{16m}\tilde{p}\rho C_{NPA}\,\alpha - \frac{D}{V}\tilde{p}\,\tilde{w}\tan|\theta| - D\tilde{r} \tag{8}$$

$$\tilde{w}' = \frac{D}{V}g\cos\theta - D^3\frac{\pi}{8m}\rho\,C_{NA}\left(\tilde{w}-\tilde{w}_W\right) - D^4\frac{\pi}{16m}\tilde{p}\rho C_{NPA}\,\beta + D\tilde{q} + \tan|\theta|\frac{D}{V}\tilde{p}\,\tilde{v} \tag{9}$$

$$\tilde{p}' = D^5\frac{\pi}{16I_{XX}}\tilde{p}\rho C_{LP} \tag{10}$$

$$\begin{aligned}\tilde{q}' = {} & D^3\frac{\pi}{8I_{YY}}\rho\,C_{NA}\left(\tilde{w}-\tilde{w}_W\right)LE_{MCP} + D^4\frac{\pi}{16I_{YY}}\rho C_{YPA}\,\tilde{p}\left(\frac{\tilde{v}-\tilde{v}_W}{V}\right)LE_{MCM} + \\ & + D^5\frac{\pi}{16I_{YY}}\rho C_{MQ}\,\tilde{q} + D^4\frac{\pi}{8I_{YY}}\rho C_{MA} - \frac{D}{V}\tilde{r}\frac{I_{XX}}{I_{YY}}\tilde{p} - \frac{D}{V}\tilde{r}^2\tan|\theta|\end{aligned} \tag{11}$$

$$\begin{aligned}\tilde{r}' = {} & -D^3\frac{\pi}{8I_{YY}}\rho\,C_{NA}\left(\tilde{v}-\tilde{v}_W\right)LE_{MCP} + D^4\frac{\pi}{16I_{YY}}\tilde{p}\rho C_{YPA}\left(\frac{\tilde{w}-\tilde{w}_W}{V}\right)LE_{MCM} + \\ & + D^5\frac{\pi}{16I_{YY}}\rho C_{MQ}\,\tilde{r} - D^4\frac{\pi}{8I_{YY}}\rho C_{MA} + \frac{D}{V}\tilde{p}\tilde{q}\frac{I_{XX}}{I_{YY}} + \frac{D}{V}\tilde{q}\tilde{r}\tan|\theta|\end{aligned} \tag{12}$$

The projectile dynamics trajectory model consists of twelve highly first order non-linear ordinary differential equations, which are solved simultaneously by resorting to numerical integration using the fourth order Runge-Kutta method. In these equations, the following sets of simplifications are employed: the aerodynamic angles of attack α and sideslip β are small $\alpha \approx \tilde{w}/V$, $\beta \approx \tilde{v}/V$, the projectile is geometrically symmetrical $I_{XY} = I_{YZ} = I_{XZ} = 0$, $I_{YY} = I_{ZZ}$ and aerodynamically symmetric. With the afore-mentioned assumptions the expressions of the distance from the center of mass to both the standard aerodynamic and Magnus centers of pressure are simplified.

Aerodynamic model

For the projectile trajectory analysis, a constant flight dynamic model is proposed for the examined test cases. The above calculations are based on appropriate constant mean values of the experimental average aerodynamic coefficients variations taken from an official tabulated database.[1] as shown in Table 2.

Initial spin rate estimation

In order to have a statically stable flight projectile trajectory motion, the initial spin rate $\tilde{p}_0$ prediction at the gun muzzle in the firing site is very important. According to McCoy's definitions,[1] the following form is used:

$$\tilde{p}_o = 2\pi V_o / \eta\, D \text{ (rad/s)} \tag{13}$$

where V_o is the initial firing velocity (m s^{-1}), η the rifling twist rate at the gun muzzle (calibers per turn), and D the reference diameter of the projectile type (m). Typical values of rifling η are 1/18 calibers per turn for big projectiles and 12 inches per turn for small bullets, respectively.

Static or gyroscopic stability

Any spinning object will have gyroscopic properties. In a spin stabilized projectile, the center of pressure, the point at which the resultant air force is applied, is located in front of the center of gravity. Hence, as the projectile leaves the muzzle it experiences an overturning movement caused by air forces acting about the center of mass. It must be kept in mind that the forces are attempting to raise the projectile's axis of rotation.

In Figure 4 two cases of static stability are demonstrated: In the top figure, CP lies behind the CG so that a clockwise (restoring) moment is produced. This case tends to reduce the yaw angle and return the body to its trajectory, therefore is statically stable. Conversely, the lower figure, with CP ahead of CG, produces an anti-clockwise (overturning) moment which increases further and is therefore statically unstable. It also possible to have a neutral case in which CP and CG are coincident whereby no moment is produced.

There is clearly an important correspondence in the distance between the center of pressure and the center of gravity and the static stability of the round. This distance is called the static margin. By definition, it is positive for positive static stability, zero for neutral stability and negative for negative stability.

Classical exterior ballistics[11] defines the gyroscopic stability factor S_g in the following generalized form:

$$S_g = \frac{I_X{}^2 \tilde{p}^2}{2\, \rho\, I_Y\, S\, D V^2 C_{MA}} \tag{14}$$

Table 2. *Aerodynamic parameters of atmospheric flight dynamic model.*

Aerodynamic coefficients		105 mm HE M1 projectile	7.62 mm bullet
Drag	C_D	0.243	0.235
Lift	C_L	1.76	2.205
Roll damping	C_{LP}	−0.0108	−0.01
Pitch damping	C_{MQ}	−9.300	−4.7
Overturning moment	C_{MA}	3.76	2.92
Magnus moment	C_{YPA}	−0.381	−1.26
Magnus force	C_{NPA}	0.215	0

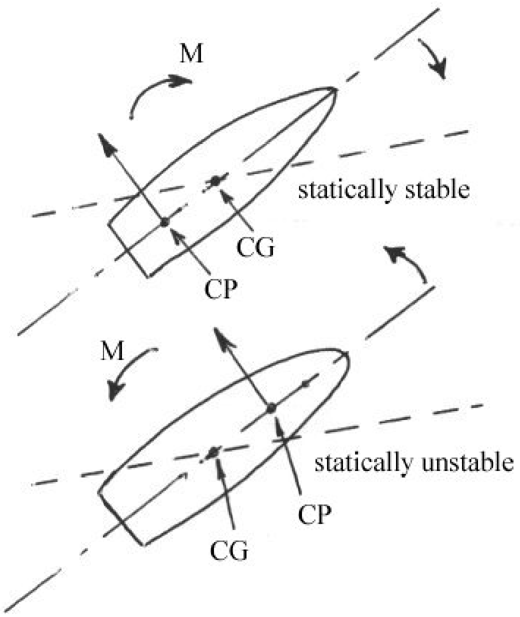

Figure 4. *Static stability/instability conditions.*

This may be rearranged into:

$$S_g = \left(\frac{2 I_X^{\,2}}{I_Y\, \pi\, D^3}\right)\left(\frac{\tilde{p}^2}{V^2}\right)\left(\frac{1}{C_{MA}}\right)\left(\frac{1}{\rho}\right) \qquad (15)$$

Equation (15) above shows that the static factor is proportional to the product of four terms, depending on the geometric technical characteristics of projectile shape model, the square axial spin to velocity ratio, the aerodynamic overturning moment coefficient and the proposed atmospheric density model.

Computational simulation

The flight dynamic model of 105 mm HE M1 and 7.62 mm projectile types involves the solution of the set of the twelve non-linear first order ordinary differential, equations (1)–(12), which are solved simultaneously by resorting to numerical integration using a fourth order Runge–Kutta method.

Initial flight conditions for both dynamic trajectory projectile models with constant aerodynamic coefficients are illustrated in Table 3.

Results and discussion

The flight path trajectory motions with constant aerodynamic coefficients of the big 105 mm projectile with initial firing velocity of 494 m s^{-1}, initial yaw angle 3°, rifling twist 1/18 caliber per turn and initial yaw rates 3.61 rad s^{-1} and 3.64 rad s^{-1} at 45° and 70°, respectively, are indicated in Figures 5 and 6. The calculated impact points of the above no-wind trajectories with the proposed constant aerodynamic coefficients compared with accurately estimations of McCoy flight trajectory analysis[1] provide basic differences for the main part of the atmospheric flight motion for the same initial flight conditions. Comparative computed trajectories of the 105 mm projectile with a 5.0 m s^{-1} mean crosswind blowing are also indicated in the above diagrams.

For the 105 mm M1 projectile, fired at 45° at sea-level neglecting wind conditions, the

Table 3. *Initial flight parameters of the projectile examined test cases.*

Initial flight data	105 mm HE M1 projectile	7.62 mm bullet
x/m	0.0	0.0
y/m	0.0	0.0
z/m	0.0	0.0
ϕ/deg	0.0	0.0
θ/deg	45 and 70	0.84 and 32
ψ/deg	3.0	2.0
u/m s^{-1}	494.0	792.48
v/m s^{-1}	0.0	0.0
w/m s^{-1}	0.0	0.0
p/rad s^{-1}	1644.0	16335.0
q/rad s^{-1}	3.61 and 3.64	25.0

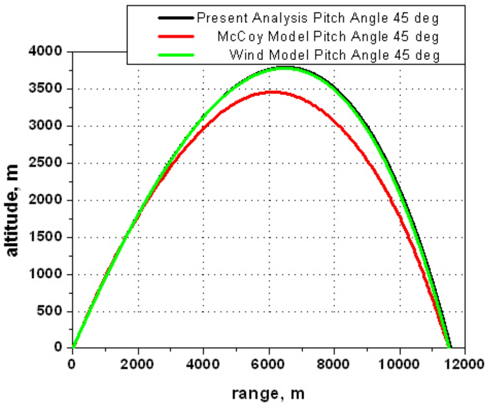

Figure 5. *Impact points with constant aerodynamic coefficients for 105 mm projectile at quadrant elevation angle of 45 degrees.*

predicted range to impact of the McCoy model is approximately 11 500 m and the maximum height is 3490 m, as shown in Figure 5. The corresponding maximum range of the wind trajectory (green solid line) is approximately 11 510 m with 3780 m maximum height. From the computational results of our presented analysis at 45°, the predicted range to impact is almost 11 550 m and the maximum height is slightly over 3795 m.

Also in Figure 6 the predicted level-ground range of the McCoy model is 7300 m at 70° with a maximum height at about 6 km. For the crosswind trajectory estimation the corresponding values are 5400 m and 6350 m, respectively. The presented method gives overestimated values of range impact and flight height calculations.

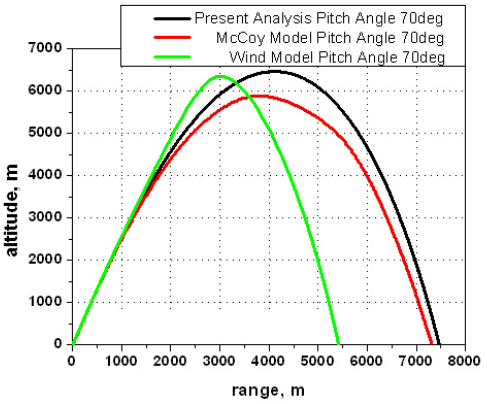

Figure 6. *Flight path trajectories with constant aerodynamic coefficients at quadrant elevation angle of 70 degrees for 105 mm projectile.*

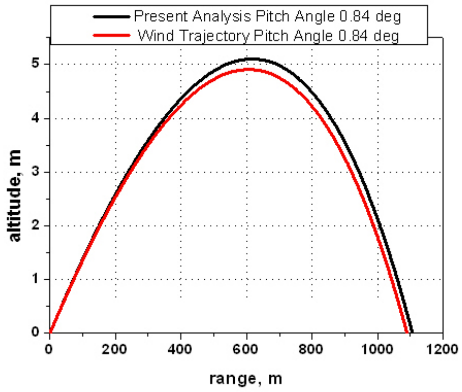

Figure 7. *Constant aerodynamic coefficients impact points for 7.62 mm bullet.*

A small bullet of 7.62 mm diameter is also examined for its atmospheric constant flight trajectory predictions in Figures 7 and 8 at low and high pitch angles 0.84° and 32°, respectively, with initial firing velocity of 793 m s^{-1}, initial yaw angle 2°, yaw rate 25 rad s^{-1} and rifling twist 12 inches per turn. The impact points of the above trajectories are compared with a corresponding 5.0 m s^{-1} mean crosswind blowing motion and an accurate flight path prediction with Nennstiel's trajectory analysis[11] for a cartridge 7.62 mm ball M80 bullet type with initial firing velocity of 838 m s^{-1}. The main differences are presented at high altitudes from the firing site sea-level.

At 0.84° the 7.62 mm bullet fired at sea-level has a range with the wind model of almost 1090 m (red

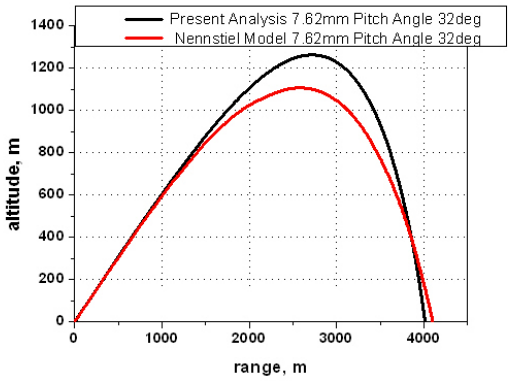

Figure 8. *Constant flight atmospheric trajectory analysis for small bullet compared to Nennstiel prediction computational algorithm.*

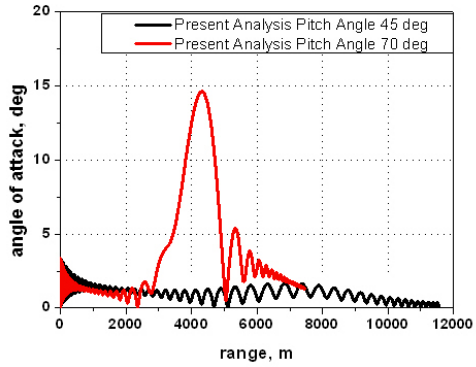

Figure 9. *Comparative angle of attack on the range of 105 mm projectile with constant coefficients at 45° and 70°.*

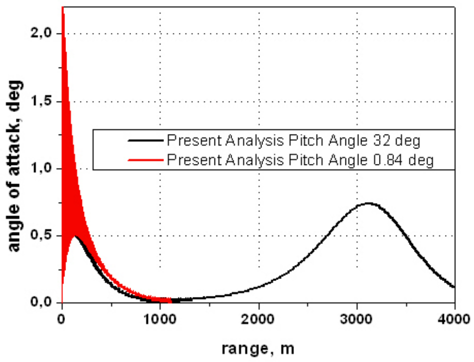

Figure 11. *Angle of attack versus range at pitch angles of 0.84° and 32° for 7.62 mm bullet.*

solid line). The presented method for the same initial pitch angle gives a corresponding value of 1105 m, and the height is slightly over 5.1 m. At 32 degrees initial pitch angle, the predicted range of the 7.62 mm bullet to the impact point of Nennstiel's analysis[11] is approximately 4100 m, and the maximum height is 1170 m. On the other hand, the proposed computational no-wind flight analysis gives an impact point at almost 4 km and a maximum height at about 1.3 km.

The strong characteristics alterations of the total angle of attack influence the distributions of the most basic projectile trajectory phenomena taking into account constant aerodynamic coefficients during the whole atmospheric flight motion. Its effects are indicated in Figure 9 for the 105 mm M1 projectile type fired from a 1/18 twist cannon with a muzzle velocity of 494 m s^{-1} at quadrant elevation angles of 45° and 70°, respectively.

After the damping of the initial transient motion, at apogee, the stability factor for 45° increases from 3.1 at the muzzle to 23, and then decreases to a value of 8 at the final impact point, as presented in Figure 10. The corresponding flight behavior at 70° initial pitch angle shows that the transient motion damps out quickly and the yaw of repose grows nearly 14° at the apogee, where the gyroscopic static stability factor has increased from 3.1 to 121 and then decreased to a value of almost 6.6 at the impact area.

Figure 11 shows the angle of attack variations with range for the 7.62 mm bullet, fired from a

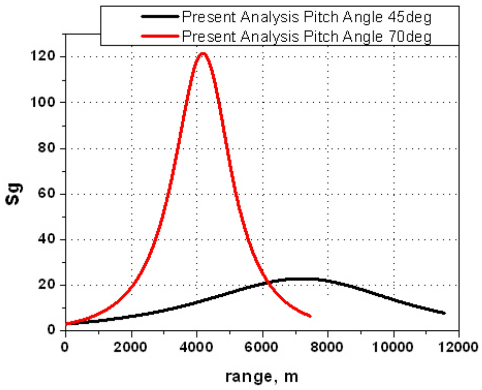

Figure 10. *Comparative static stability variation with constant aerodynamic model at low and high quadrant angles for the 105 mm projectile*

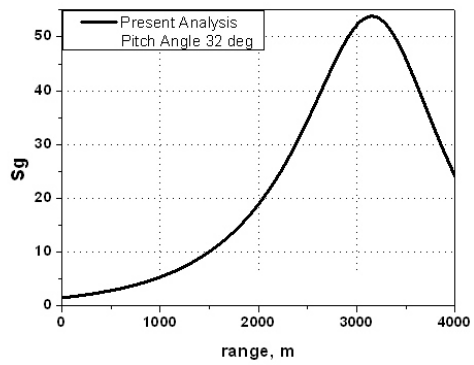

Figure 12. *Static stability versus range at pitch angle of 32°, for 7.62mm bullet.*

12 inch twist per turn with a muzzle velocity of 793 m s^{-1} at quadrant elevation angles of 0.84° and 32°, respectively. The transient motion damps out very quickly for the two cases. The gyroscopic stability factor for 32° was 1.5 at the muzzle, increased to 54 at the summit of the trajectory, and then decreased to a value of 24 at the impact point, as indicated in Figure 12.

The estimation of static stability factors, equation (14), for the examined projectile test cases is based only on the square axial spin to velocity ratio due to the constant overturning aerodynamic moment. The magnitude of this suggests that if a projectile or bullet is statically stable at the muzzle, it will be statically stable for the rest of its atmospheric flight motion. As the rotational velocity is much less damped than the transversal velocity (which is damped due to the action of the drag force), the static factor S_g increases at least for the major part of the predicted projectile trajectory.

Conclusions

The six degrees of freedom (6-DOF) simulation flight dynamics model is applied for the accurate prediction of short and long range trajectories of high and low spin-stabilized projectiles and small bullets. The parallel computational analysis takes into consideration the effects of constant aerodynamic coefficients. The criteria and analysis of gyroscopic stability are also examined. The computational results of the dynamic flight trajectories with constant aerodynamic force and moment coefficients are in good agreement compared with other technical data and recognized projectile flight dynamic models.

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