Exterior Ballistics of Firework Stars

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Abstract: *Burning and shot experiments have been carried out to examine the burning time and the trajectory of burning firework stars. The burning time in flight was longer than the stationary burning time. The trajectory of a burning star in the air was modeled assuming the linear burning rate of the star and* $C_D = at + b$. *It was found that there are sometimes abnormal trajectories and interruption of the burning during flight for a large star with high initial velocity.*

Keywords: *Exterior ballistics, firework star, burning rate, initial velocity*

Introduction

Firework stars with long burning times are used in willow and crown aerial fireworks. The exterior ballistics of burning stars of this type is important for the design of the shape of the willow and crown.

Shimizu has studied the ballistics of burning stars expelled from exploded shells.[1,2] Kosanke and Kosanke reported the partial burning of the expelled stars.[3] Kosanke and Kosanke[4] and Mercer[5] have modeled the exterior ballistics of aerial firework shells.

In the present work, measurement of the burning time of stationary and flying stars was carried out, and the trajectory of the fired stars was observed. Then, the observed data were analyzed.

Experimental

Samples

The silver crown stars for no. 2.5, 4 and 5 shells (a Japanese no. 2.5 round shell corresponds to a western 3 inch shell) were supplied by Sunaga Fireworks Company at Ashikaga-city, and the lifting charge and electric match were made by Nippon Kayaku Company.

The mortars

The two mortars used for firing stars were made of steel, with inner diameters of 15 mm and 20 mm, and depths of 270 mm and 360 mm, respectively.

Experimental method

A star was placed on a heat resistant board and ignited with a torch. The stationary burning time of the star was determined using a high-speed video camera (Phantom VR-V4.2) with a frame rate of 1000 frames s^{-1}. An electric match was placed on the bottom of the mortar, and the lifting charge was poured into the vertical mortar from the muzzle. The electric match was ignited by an electric current from a 12 V battery and then the lifting charge was ignited immediately by the match. The trigger of the high-speed video camera worked simultaneously with the turning on of the electric current.

The trajectory and burning time of the star were observed and recorded by the high-speed video camera with a frame rate of 1000 frames s^{-1} at a point 100 m away from the mortar.

Analysis of the motion of a star in the air

The motion of a burning star fired vertically in the air is expressed by following equation (1):

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -g - \frac{3\rho_{\mathrm{air}}}{4\rho_{\mathrm{star}}} \times \frac{C_{\mathrm{D}}}{D_{\mathrm{star}}} \times |u| \times u \qquad (1)$$

or

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -g - k \times |u| \times u \qquad (2)$$

Here u, t, g, ρ_{air}, ρ_{star}, D_{star} and C_{D} are the velocity of the star, flying time, acceleration under gravity, air density, density of the star, diameter of the star and drag coefficient of air, respectively. The velocity u is positive when the star moves upward.

and

$$k = \frac{3\rho_{\text{air}}}{4\rho_{\text{star}}} \times \frac{C_{\text{D}}}{D_{\text{star}}} = \frac{3\rho_{\text{air}}}{4\rho_{\text{star}}} \times K \qquad (3)$$

It was assumed that the mass and cross section of a star change but the density of the star does not change with time. Therefore, the second term on the right side of equation (1) for the air drag is proportional to the air drag coefficient C_D and inversely proportional to the diameter of the star. Finally, this term is proportional to K (= C_D/D_{star}).

The value of K was obtained by fitting the calculated trajectory to the observed one of the fired star. The diameter D_{star} of the star is a function of the flying time of the star and is calculated from the mean linear burning rate of stars. Then, C_D was calculated from K and D_{star} using equation (3).

Results and discussion

Stationary and flying burning times of stars

The stationary and flying burning times of stars are listed in Table 1.

The flying burning time was longer than the stationary burning time.[6] In the present work, this nature was confirmed. The ratio of the flying burning time to the stationary burning time (t_f/t_s) was 1.44–1.67 for stars of no. 2–6 shells in the previous work,[6] and 1.36 and 1.46 in the present work for the stars of no. 4 and no. 5 shells, respectively.

One of reasons that the flying burning time is longer than the stationary burning time may be partial burning in the flight of a star. Kosanke and Kosanke showed by means of a photograph that part of the star expelled from an exploded shell was black, indicating that part of the flying star was flameless. The flame at the front surface of the expelled star might be extinguished by the high-speed air-flow and result in a slower burning rate of the star. Another possible reason might be the cooling effect of air-flow around the burning star.

Fit of *K*

The value of K obtained by fitting the calculated trajectory to the observed one gave good agreement between the observed and calculated trajectories of vertically fired stars for 25 cases among 32 shots. However, no good agreement was obtained for 7 cases and no data for one case. Examples of good agreement and poor agreement are shown in Figure 1.

At the moment, the cases where poor agreement was obtained were the cases where the initial velocities were low and the burning stars dropped on the ground.

In the present work, most of the stars for a no. 5 shell with a 2 g lifting charge (LC) failed to continue burning until complete combustion. The interruption of burning may be attributed to the high-speed air-flow around the burning star. In the other case, the flame at the star disappeared once and then the flame appeared again in the course of flying.

Figure 2 shows some examples of the plot of calculated C_D against time for the cases where there was good agreement.

Table 1. *Stationary and flying burning time of the silver crown stars.*

		Mass/g		Diameter/mm		Burning time/s		t_f/t_s
		Mean	RSD	Mean	RSD	Mean	RSD	
Stars for no. 2.5 shell	Stationary	2.734	0.024	13.908	0.021	3.601	0.073	
	Flying	2.747	0.048	13.862	0.019			
Stars for no. 4 shell	Stationary	4.737	0.037	16.565	0.013	4.198	0.042	1.36
	Flying	4.964	0.027	16.840	0.014	5.701	0.047	
Stars for no. 5 shell	Stationary	5.592	0.029	17.636	0.014	4.646	0.043	1.46
	Flying	5.640	0.028	17.624	0.012	6.761	0.030	

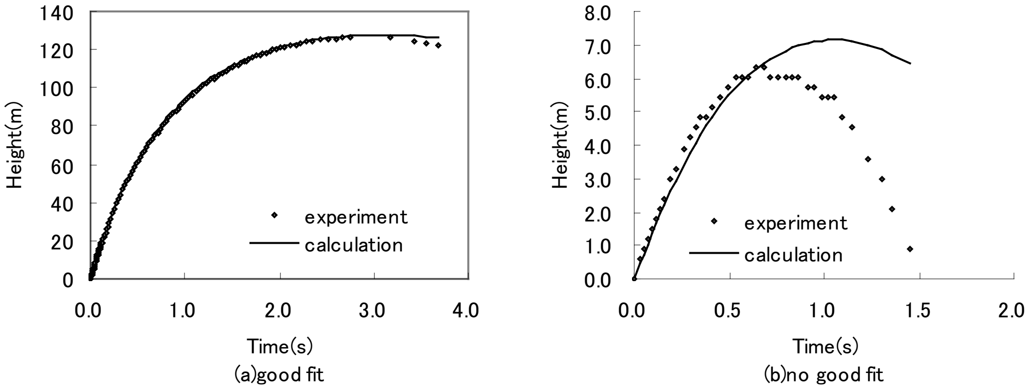

Figure 1. *Examples of plot of flying height vs. flying time of star.*

All calculated trajectories using these C_D values agreed well with the observed trajectories, but the C_D values were scattered at later times. Also two abnormal C_D values were obtained (runs 11 and 21).

Modeling of the motion of the fired star

The trajectory of a fired star in the air was modeled by following procedures:

(1) K in equation (3) is obtained as a function of time by fitting the calculated trajectory to the observed trajectory of a fired star and both trajectories are plotted in the time–height coordinates (Figure 3(a)).

(2) C_D is obtained as the function of time from K and D_{star} using equations (3) and (4) (Figure 3(b)).

$$D_{star} = D_{star0} - 2r_{star}t \qquad (4)$$

(3) C_D has little effect on the air drag when the velocity is small as shown in ref. 7. As shown in Figure 3(c), the flying velocity of the star becomes small after 2 seconds after the shot in this case.

(4) C_D is assumed to be a linear function of flying time before the star arrives at its maximum height (Figure 3(d)). In this case the following equation was obtained for approximate C_{Da}:

$$C_{Da} = 0.4589t + 0.2864 \qquad (5)$$

(5) An approximate value for K_a is calculated using equations (3), (4) and (5) (Figure 3(e)).

(6) The approximate trajectory of the burning star is estimated using K_a and the agreement between the calculated and observed trajectories is confirmed (Figure 3(f)).

Table 2 lists the conditions of the experiment and

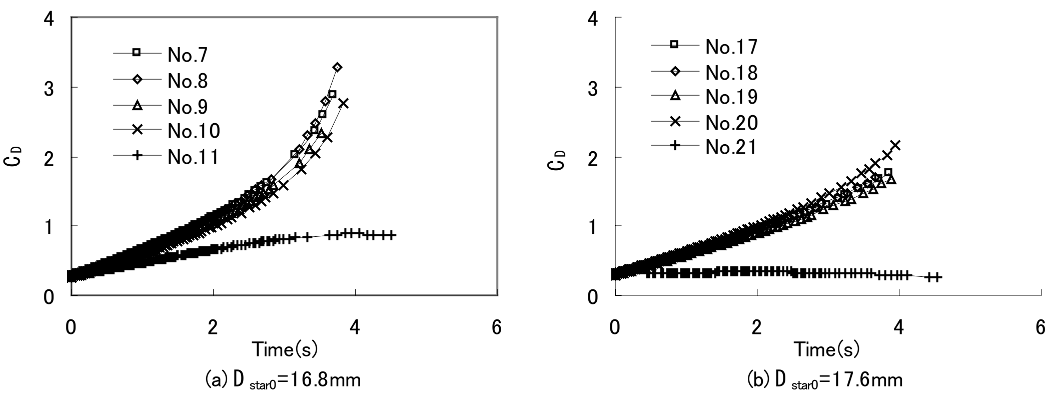

Figure 2. *Plot of calculated C_D vs. time. LC = 1.0 g.*

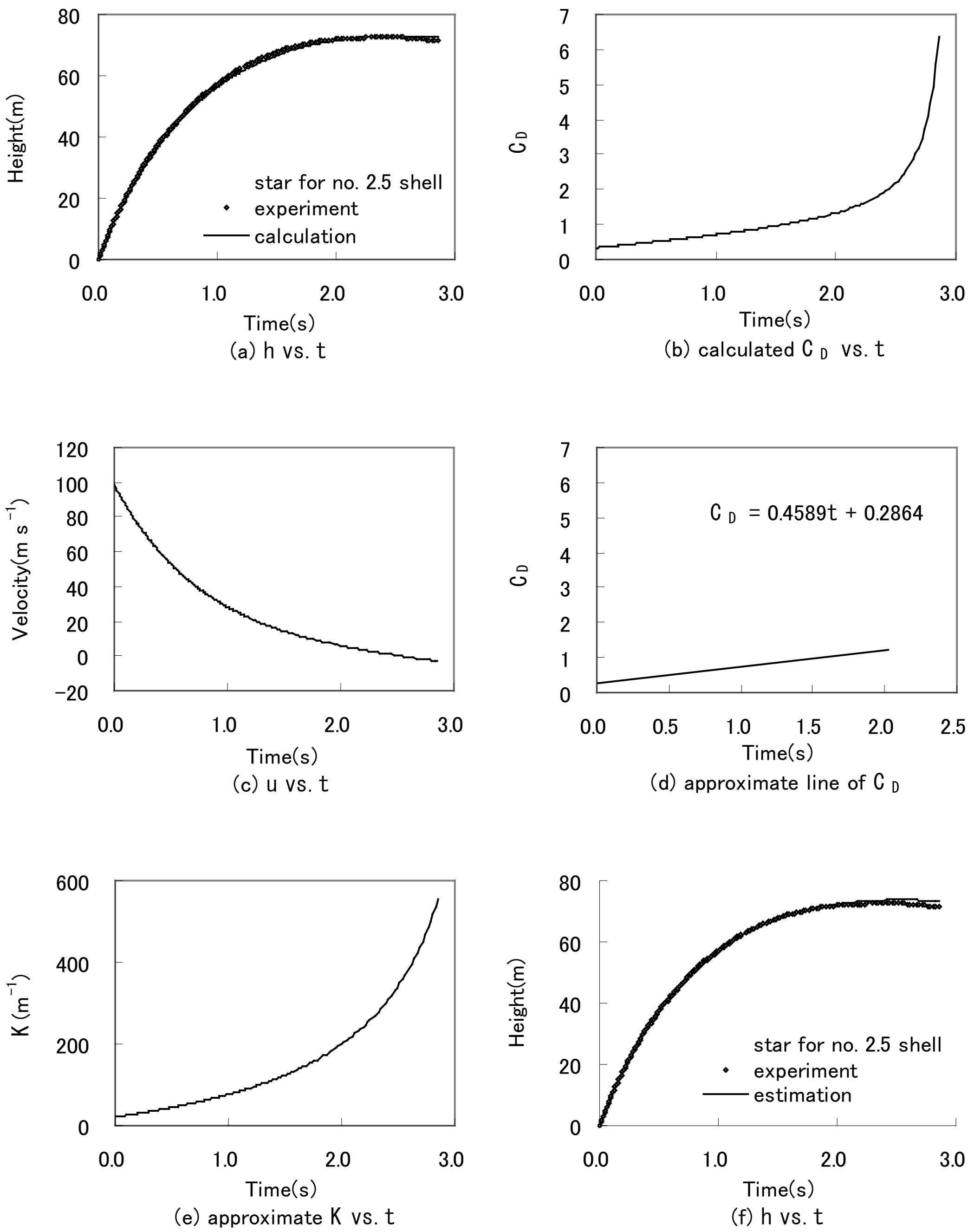

Figure 3. *Modeling of the motion of a star in the air.*

the results.

The mean values of initial diameter $\bar{D}_{\text{star}0}$, linear burning rate $\bar{r}_{\text{star}}$, and lifting charge (LC) of stars for runs 7, 8, 9, 10 and runs 17, 18, 19, 20 are listed in Table 3, respectively.

The calculated maximum height attained and the time to the maximum height of the stars using these mean parameters are listed in Table 3. The results can be compared to the observed values listed in Table 2. The calculated mean trajectory and observed trajectories are shown in Figure 4. The calculated drag coefficient C_D increased with time and the initial values were about 0.26 though there were some exceptions. At the moment, the reason for these phenomena is not clear.

Table 2. *Experimental conditions and results.*

No.	D_{star0}/ mm	LC/g	u_0/m s^{-1}	Mortar diam. /mm	Fit to *K*	$C_D = at+b$		h_{max}/m		t_{max}/s	
						a	*b*	Obs.	Calc.	Obs.	Calc.
1	13.4	1.00	98	15	yes	0.4589	0.2864	72.5	73.7	2.490	2.529
2	13.8	1.00	15	15	no						
3	14.2	1.00	31	15	no						
4	14.0	1.00	218	15	yes	0.0574	0.3032	162.7	160.7	3.792	3.702
5	14.0	1.00	36	15	no						
6	13.8	2.00		15	no data						
7	16.8	1.00	168	20	yes	0.4267	0.2546	126.3	126.8	2.760	2.964
8	16.4	1.00	168	20	yes	0.4143	0.2599	125.7	127.4	2.807	3.002
9	16.8	1.00	156	20	yes	0.3782	0.2619	121.5	123.3	2.782	3.024
10	17.0	1.00	144	20	yes	0.36	0.2592	121.5	118.0	2.975	3.109
11	16.9	1.00	168	20	yes	0.1909	0.2688	145.9	145.3	3.313	3.469
12	17.3	2.00	217	20	yes	0.2805	0.2374	163.6	165.1	3.185	3.390
13	17.0	2.00	217	20	yes	0.1896	0.3047	159.1	158.5	3.393	3.505
14	16.7	2.00	241	20	yes	0.1044	0.3066	*170.8*[a]	185.1	*2.357*	3.782
15	16.8	2.00	205	20	yes	0.1069	0.3494	155.2	155.5	3.410	3.571
16	16.7	2.00	265	20	no						
17	17.3	1.00	165	20	yes	0.3442	0.2699	130.8	131.6	2.972	3.208
18	17.3	1.00	158	20	yes	0.3485	0.2845	126.9	127.6	3.124	3.190
19	17.6	1.00	165	20	yes	0.321	0.2823	135.6	134.4	3.228	3.278
20	17.6	1.00	180	20	yes	0.396	0.2632	135.9	137.6	3.176	3.254
21	17.9	1.00	165	20	yes	0.0208	0.3017	*171.4*[a]	172.2	*4.104*	4.266
22	17.5	2.00	256	20	yes	0	0.15	*22.3*[b]	365.6	*0.092*	5.980
23	18.0	2.00	263	20	yes	0	0.15	*21.7*[b]	364.9	*0.088*	6.016
24	17.6	2.00	218	20	yes	0	0.15	*20.5*[b]	332.8	*0.100*	5.900
25	17.6	2.00	256	20	yes	0.0822	0.3242	170.2	192.9	1.956	3.876
26	17.6	2.00	271	20	yes	0	0.15	*20.2*[b]	365.7	*0.080*	5.840
27	17.9	0.25	60	20	no						
28	17.3	0.50	90	20	no						
29	17.8	0.75	135	20	yes	0.4597	0.2359	115.2	113.5	2.896	3.024
30	17.6	1.25	180	20	yes	0.244	0.2836	149.5	148.9	3.540	3.378
31	17.8	1.50	226	20	yes	0.0097	0.3193	*171.1*[a]	201.9	*2.112*	4.312
32	17.5	1.75	248	20	yes	0.0125	0.3788	*170.8*[a]	189.0	*2.488*	4.056

Note: The data in italics are not really the maximum height and the time to maximum height of a star because (a) it went out of sight of the camera before it arrived at its maximum height or (b) its burn was interrupted and the rest of the trajectory could not be recorded by the camera.

Abnormal and interrupted burning trajectories

Sometimes abnormal trajectories of the burning stars were observed. For example, the trajectories of runs 11 and 21 were different from other groups of the same initial conditions. In the equation

$$C_D = at + b,$$

the values of *a* are different from other groups, though the values of *b* are similar to those of other groups. In reality, abnormal flights occur sometimes.

The other characteristics of the abnormal

Table 3. *Experimental conditions, mean calculated parameters, and calculated and observed trajectories of the burning stars.*

No.	D_{star0} /mm	r/mm s^{-1}	u_0/m s^{-1}	LC/g	$C_D = at + b$		h_{max}/m	t_{max}/s
					a	b	Calc.	Calc.
Runs 7–10	16.8	1.49	159	1.0	0.395	0.259	129.1	3.163
Runs 17–20	17.5	1.32	167	2.0	0.352	0.275	137.4	3.167

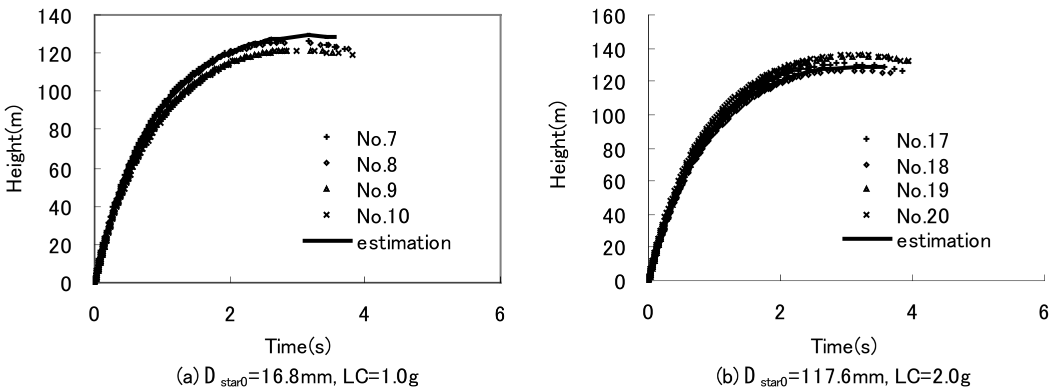

Figure 4. *Calculated mean trajectory and observed trajectories of burning stars with (a) D_{star0} = 16.8 mm and LC = 1.0 g, (b) D_{star0} =17.6 mm and LC = 2.0 g.*

trajectories are higher maximum height attained and longer time required to attain maximum height. These phenomena may be attributable to the slower burning speeds of stars with abnormal trajectories than those of normal ones.

Also, interruption of burning took place with large stars (for no. 5 shells) and large lifting charges (2.0 g). It is known that the flames of stars expelled from a burst shell are sometimes partly extinguished.[3] In our case, the flames of the stars were blown out completely.

In the case of the blown-out flames, the values of *a* and *b* in the C_D equation are both smaller than those in normal flight. The shape of the flame of the burning stars in the interrupted case might be different from that of normal burning stars.

Effect of the amount of lifting charge on the trajectories

The amount of lifting charge affects the parameters *a* and *b*. The value of *a* decreases and that of *b* increases with increasing amount of lifting charge. With the stars of no. 5 shells, burning of the stars was interrupted with 2.0 g LC in which the *a* values were much smaller than those of normal cases.

Conclusions

(1) The burning time of a star in flight is longer than the stationary burning time, presumably because of the cooling effect of the flowing air around the star.

(2) The trajectory of a burning star in the air was modeled assuming a linear burning rate of the star and $C_D = at + b$. This method is not valid for the falling trajectory.

(3) It was found that there are sometimes abnormal trajectories and interruption of burning during flight for high initial velocity and large stars.

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