

Computer Modeling of Flying Star Ballistics

Dayu Ding, Morimasa Higaki, Yozo Ooki and Tadao Yoshida*

Ashikaga Institute of Technology,
268-1 Omae-cho, Ashikaga-shi, Tochigi 326-8558, KAPAN
Tel: +81-284-62-0605
Fax: +81-284-62-0976
E-mail yoshida@ashitech.ac.jp

*To whom all correspondence should be addressed.

Abstract: *The burning time of stationary and flying fireworks stars was measured, the trajectories of flying stars were observed and the results were analyzed in this work. It was found that the difference in burning time between the stationary and flying burning stars was dependent on the kind of star. Modeling of flying star ballistics was applied to the trajectory of stars with shorter burning times and was found valid for this case.*

Keywords: *fireworks, burning time, exterior ballistics, modeling*

Introduction

Firework stars with short burning times are used for warimono aerial shells giving spherical fire flowers, katamono aerial shells giving various burst shapes, Roman candles, and so on. The ballistics of stars with short burning times are important for designing warimono and katamono shells, Roman candles and others.

The present authors have suggested a computer model for estimating the trajectory of the burning star expelled from a mortar using a star with longer burning time.¹ In the present work, the model is applied to a flying star with a shorter burning time and the results are analyzed.

Computer modeling of aerial shells has been done by Kosanke and Kosanke,² and Mercer.³ In the case of an aerial shell, the drag coefficient (C_D) can be assumed constant before the bursting of the shell in the air. But, in the case of a burning star, the above assumption is not valid, and an approximate approach was adopted in our previous work.¹

Experimental

Materials

The silver peony stars for no. 2, 2.5, 3, 4 and 5 shells (a Japanese no. 2.5 shell corresponds to a Western 3 inch shell), lifting charge and electric matches were supplied by Sunaga Fireworks Co. Ltd. at Ashikaga City.

Grain black powder made by Nippon Kayaku Co. Ltd. was used as the lifting charge in our experiments. The standard and particle distribution of the lifting charge were described in our previous paper.⁵

Apparatus

The three kinds of mortar used for firing stars were made of steel, and the inner diameters were 12 mm, 15 mm and 20 mm, and depths were 216 mm, 270 mm and 360 mm, respectively.

The burning time and trajectories of stars were measured and recorded using a high-speed video camera (Phantom VR-V4.2).

Procedure

In stationary burning experiments, a star was placed on a heat resistant board and ignited by a torch. The stationary burning time of the star was determined using a high-speed video camera.

In star shooting experiments, the mortar was set on the ground vertically. An electric match was placed on the bottom of the mortar, the lifting charge was poured into the mortar from the muzzle, and then a star was placed on the lifting charge. The electric match was ignited by turning on an electric current. The star was shot into the air after the lifting charge burned.

The trajectory of the star was recorded by the high-speed camera. Each frame of the video was

reproduced on a video screen, and the burning time and the relationship between flying time and flying height of the star were determined.

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Theory

The motion of a burning star expelled vertically in the air can be expressed as follows.

$$\frac{du}{dt} = -g - \frac{3\rho_{\text{air}}}{4\rho_{\text{star}}} \cdot \frac{C_D}{D_{\text{star}}} \cdot |u| \cdot u \quad (1)$$

or

$$\frac{du}{dt} = -g - \frac{3\rho_{\text{air}}}{4\rho_{\text{star}}} \cdot K \cdot |u| \cdot u \quad (2)$$

Here, u , t , g , ρ_{air} , ρ_{star} , D_{star} and C_D are star velocity, time, acceleration of gravity, air density, star density, diameter of the star and drag coefficient of air, respectively. The velocity u is positive when the star moves upward.

And

$$K = \frac{C_D}{D_{\text{star}}} \quad (3)$$

It is assumed that the mass and cross sectional area of a burning star change but the density of the star does not change with time. Therefore, the second term of the right hand side of equation (1) for the air drag is proportional to the air drag coefficient C_D and inversely proportional to the diameter of the star D_{star} as expressed in equation (3).

K is obtained by the step by step calculation of the increment of the trajectory using equation (2). The diameter D_{star} of the star is a function of the flying time of the star and is calculated from the linear burning rate of the star:

$$D_{\text{star}} = D_{\text{star}0} - 2r_{\text{star}}t \quad (4)$$

Here, $D_{\text{star}0}$ and r_{star} are the initial diameter and the linear burning rate of the star, respectively.

Then, C_D is calculated from K and D_{star} using equations (3) and (4). It was found that C_D is nearly a linear function of time in the earlier stages of the trajectory and a scattered complex function of time in the later stages.¹ It was also found that in the low velocity range C_D of firework shells has little effect on the air drag.⁴

In our modeling of flying star ballistics, C_D is approximated as the linear function of time and expressed in equation (5):

$$C_D = at + b \quad (5)$$

a and b in equation (5) are determined from the star shooting experiments and are used for estimating the trajectory of a burning star.

Results and Discussion

Stationary and flying burning times of stars

The mean burning time of the stationary and flying stars is listed in Table 1. In the previous experiments,^{1,6} t_f/t_s were much larger than 1.0, but in this experiment, values of t_f/t_s were nearly 1.0. The differences may be attributable to the nature of the stars used.

Experimental and calculated results of the trajectories of stars

The experimental and calculated results of the trajectories of stars are listed in Table 2. The relative standard deviations of observed data were below 10% except for the initial velocity of the stars. The absolute values of the relative standard deviation of a are large because the mean values of a are very small. In this case the relative standard deviation has less meaning.

Fit of the calculated to the observed trajectory

All of the calculated trajectories were fitted to the observed ones. Examples are shown in Figure 1.

Effect of the kind of stars on a and b

In our previous work,¹ different values of a and b in equation (5) were obtained with the silver crown stars for no. 4 and no. 5 shells compared to those with silver peony stars in this work as listed in Table 3. The burning behavior may be different with different kinds of stars.

Table 1 Mean burning times of the stationary and flying stars.

Star	Stationary				Flying				t_f/t_s
	D_{star0} (mm)	Burning time(t_s) (ms)	SD	RSD	D_{star0} (mm)	Burning time (t_f) (ms)	SD	RSD	
Silver peony for									
No. 2 shell	10.27	2608	163	0.06	10.33	2751	126	0.05	1.05
No. 2.5 shell	11.19	2833	42	0.01	11.29	3311	197	0.06	1.17
No. 3 shell	12.13	3340	115	0.03	12.38	3389	318	0.09	1.01
No. 4 shell	13.30	3659	102	0.03	13.37	3464	153	0.04	0.95
No. 5 shell	14.91	3809	828	0.22	15.01	3967	138	0.03	1.04
No. 5 shell*	14.38	3822	195	0.05					1.00
Silver crown for									
No. 4 shell	16.57	4198		0.04	16.84	5701		0.05	1.4 ¹
No. 5 shell	17.64	4646		0.04	17.62	6761		0.03	1.5 ¹
Blue peony, silver peony and silver crown for									
No. 2–6 shells									1.6 ⁶

*Half the ignition promoter was scraped off and covered by an inhibitor. SD is standard deviation, and RSD (=SD/mean) is relative standard deviation.

Effect of the size and initial velocity of stars on a and b

The calculated results for stars from no. 2 (smallest) and no. 5 (largest) shells are shown in Figure 2. Values of C_D for the smallest star from a no. 2 shell increased with time and C_D of the largest from a no. 5 shell decreased with time. b in equation (5) was 0.46–0.53 with the smallest star from a no. 2 shell, while it was 0.55–0.63 for the largest star from a no. 5 shell.

Plots of a and b against the initial size and velocity of the stars are shown in Figure 3. The scatter of a of the stars for a no. 2 shell (runs 1–5) is abnormally large and the value for run 19 is also abnormally small. These data were excluded from the statistical consideration.

The correlations of a and b with D_{star0} and u_0 do not appear significant. Therefore, the mean values of a and b calculated from original data were -0.010 and 0.57 , respectively.

The estimated trajectories of the burning silver peony stars using the above values of a and b are plotted against time in Figure 4 along with the observed trajectories.

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Table 2 Experimental and calculated results on the trajectories of the silver peony stars.

Run No.	ID of mortar (mm)	Lift charge (g)	Shell	Mass (g)	D_{star0} (mm)	u_0 (m s^{-1})	Burning time t_f (ms)	r_{star} (mm s^{-1})	$C_D = at + b$	
									a	b
1	12	0.5	No. 2	1.12	10.2	81		2.12	0.069	0.526
2	12	0.5	No. 2	1.05	10.2	61	2772	1.95	0.056	0.455
3	12	0.5	No. 2	0.94	9.8	61	2736	1.86	0.036	0.492
4	12	0.5	No. 2	1.16	10.9	62	2594	2.12	-0.001	0.528
5	12	0.5	No. 2	1.18	10.5	85	2901	1.87	0.036	0.512
Mean				1.09	10.3	70	2751	1.99	0.039**	0.503
SD				0.10	0.4	12	126	0.13	0.026	0.030
RSD				0.09	0.04	0.17	0.05	0.07	0.669	0.060
6	12	0.5	No. 2.5	1.46	11.3	87	3041	1.85	-0.001	0.490
7	12	0.5	No. 2.5	1.45	11.3	92	3478	1.62	-0.003	0.510
8	12	0.5	No. 2.5	1.40	11.2	100	3436	1.63	-0.006	0.571
9	12	0.5	No. 2.5	1.52	11.7	80	3288	1.78	-0.012	0.531
10	12	0.5	No. 2.5	1.36	11.0	86		1.74	-0.004	0.530
Mean				1.44	11.3	89	3311	1.73	-0.005	0.526
SD				0.06	0.3	8	197	0.10	0.004	0.030
RSD				0.04	0.02	0.09	0.06	0.06	-0.835	0.057
11	15	1	No. 3	2.02	12.6	76		2.30	0.000	0.610
12	15	1	No. 3	1.95	12.5	87	3808	1.85	-0.009	0.561
13	15	1	No. 3	1.90	12.5	93	3110	2.10	0.000	0.490
14	15	1	No. 3	1.89	12.3	95	3462	2.15	0.000	0.530
15	15	1	No. 3	1.85	12.1	94	3176	2.04	-0.008	0.571
Mean				1.92	12.4	89	3389	2.09	-0.004	0.552
SD				0.06	0.2	8	318	0.16	0.005	0.045
RSD				0.03	0.01	0.09	0.09	0.08	-1.327	0.082
16	15	1	No. 4	2.43	13.1	111	3300	2.09	-0.033	0.624
17	15	1	No. 4	2.61	13.6	86	3604	1.99	-0.014	0.612
18	15	1	No. 4	2.49	13.6	102		2.13	-0.005	0.601
19	15	1	No. 4	2.38	13.3	108	3488	2.15	-0.071*	0.595
20	15	1	No. 4	2.34	13.3	103		1.84	-0.003	0.590
Mean				2.45	13.4	102	3464	2.04	-0.014	0.604
SD				0.10	0.2	10	153	0.13	0.028	0.014
RSD				0.04	0.02	0.10	0.04	0.06	-1.110	0.023
21	20	2	No. 5	3.46	15.2	79	3950	2.20	-0.013	0.612
22	20	2	No. 5	3.36	14.7	91	3916	2.01	-0.015	0.592
23	20	2	No. 5	3.58	15.5	115	4198	2.01	-0.014	0.611
24	20	2	No. 5	3.34	14.7	88	3826	2.03	-0.024	0.635
25	20	2	No. 5	3.46	15.1	92	3944	2.04	-0.019	0.553
Mean				3.44	15.0	93	3967	2.06	-0.017	0.600
SD				0.10	0.3	13	138	0.08	0.005	0.031
RSD				0.03	0.02	0.14	0.03	0.04	-0.269	0.051
Overall mean									-0.010	0.560

* These data are omitted from the mean calculation. ** These data are omitted from the overall mean calculation.

Table 3 Values of a and b in equation (5) for different stars.

Kind of star	Mean diameter (mm)	Mean u_0 (mm s ⁻¹)	a	b
Silver crown star for				
No. 4 shell	16.8	159	0.359	0.259
No. 5 shell	17.5	167	0.352	0.275
Silver peony star for				
No. 4 shell	13.4	102	0.014	0.604
No. 5 shell	15.0	93	0.017	0.600

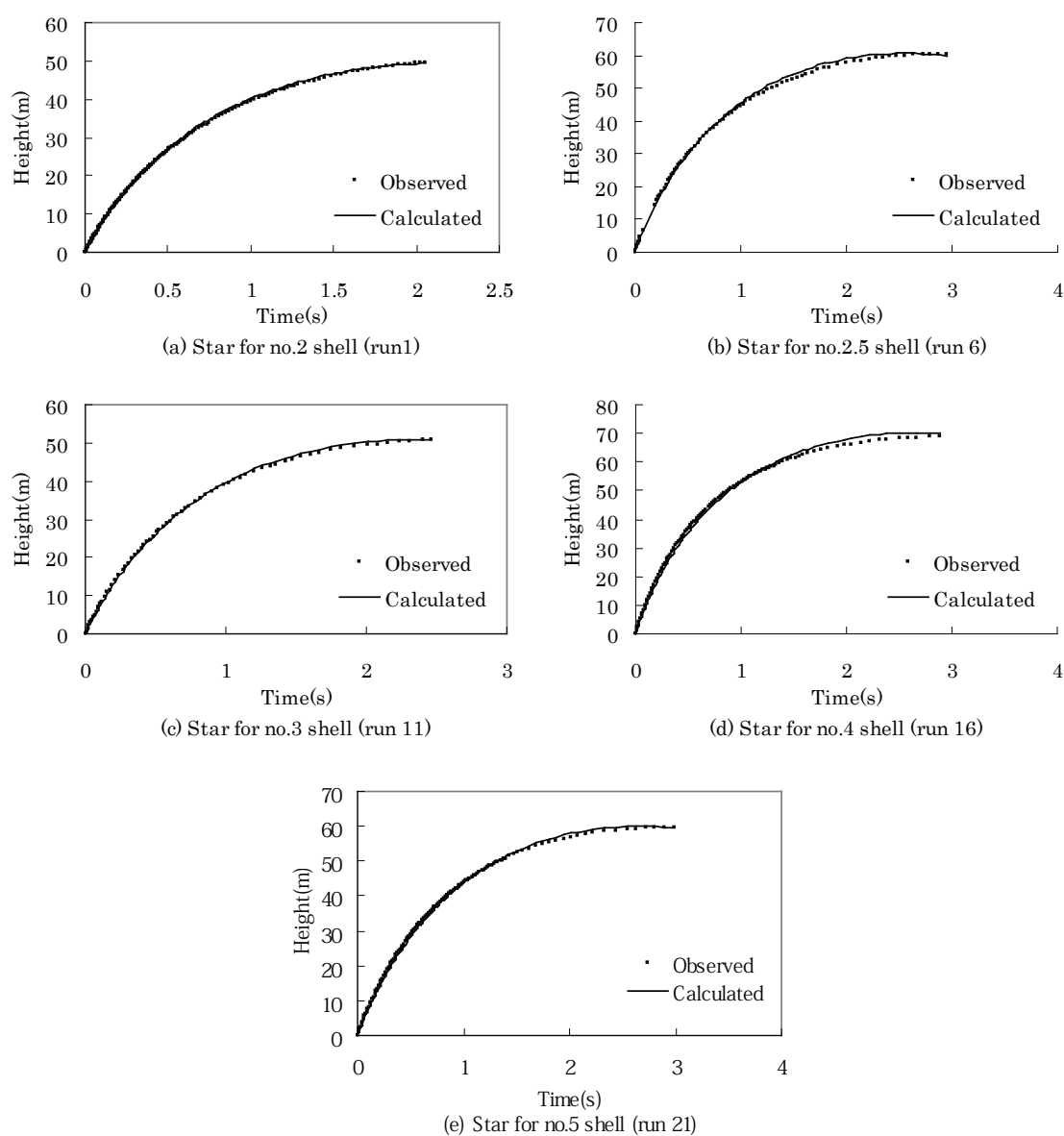


Figure 1 Examples of the observed and calculated trajectories of burning stars.

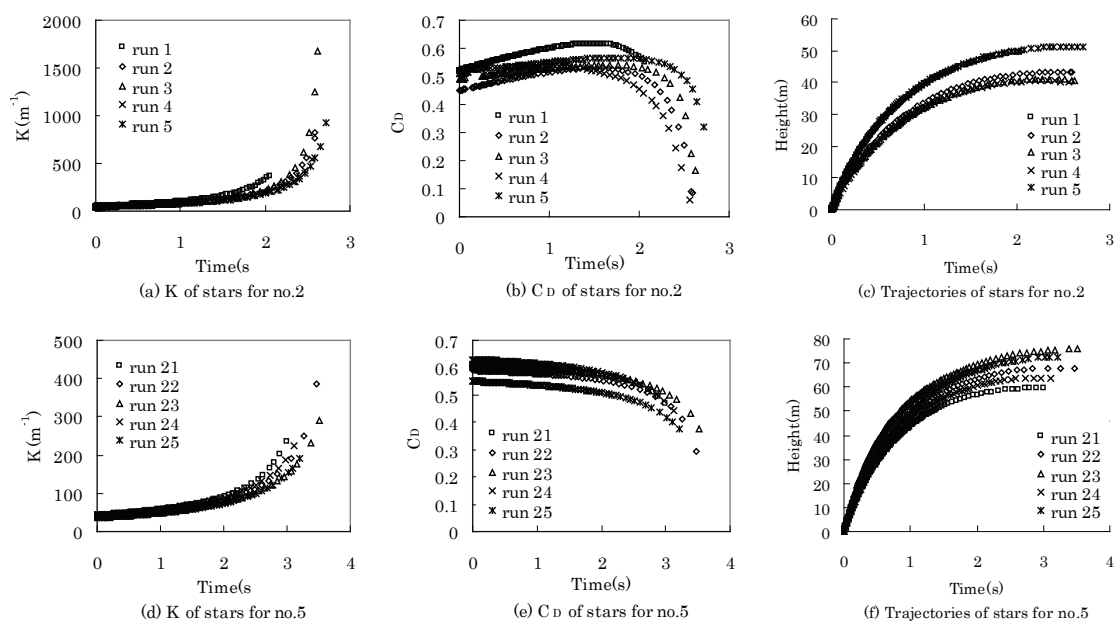


Figure 2 Calculated K , C_D and flying height vs. flying time for no. 2 and no. 5 shells.

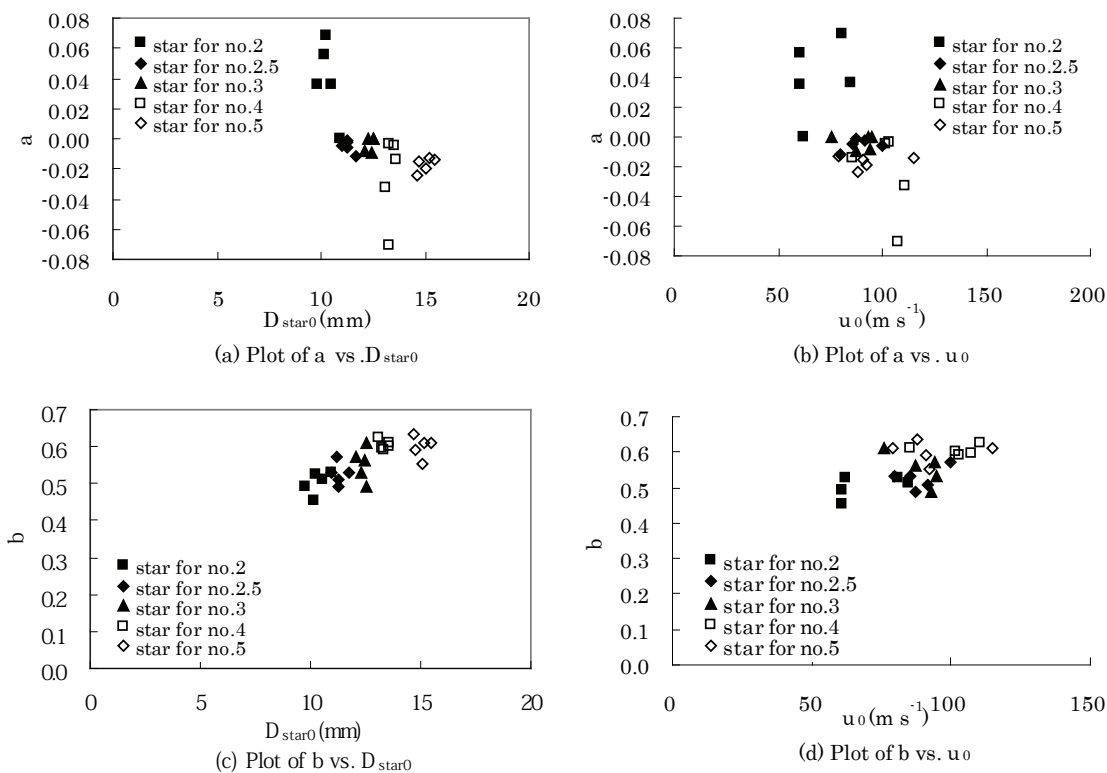


Figure 3 Plots of a and b vs. the initial size and velocity of burning silver peony stars.

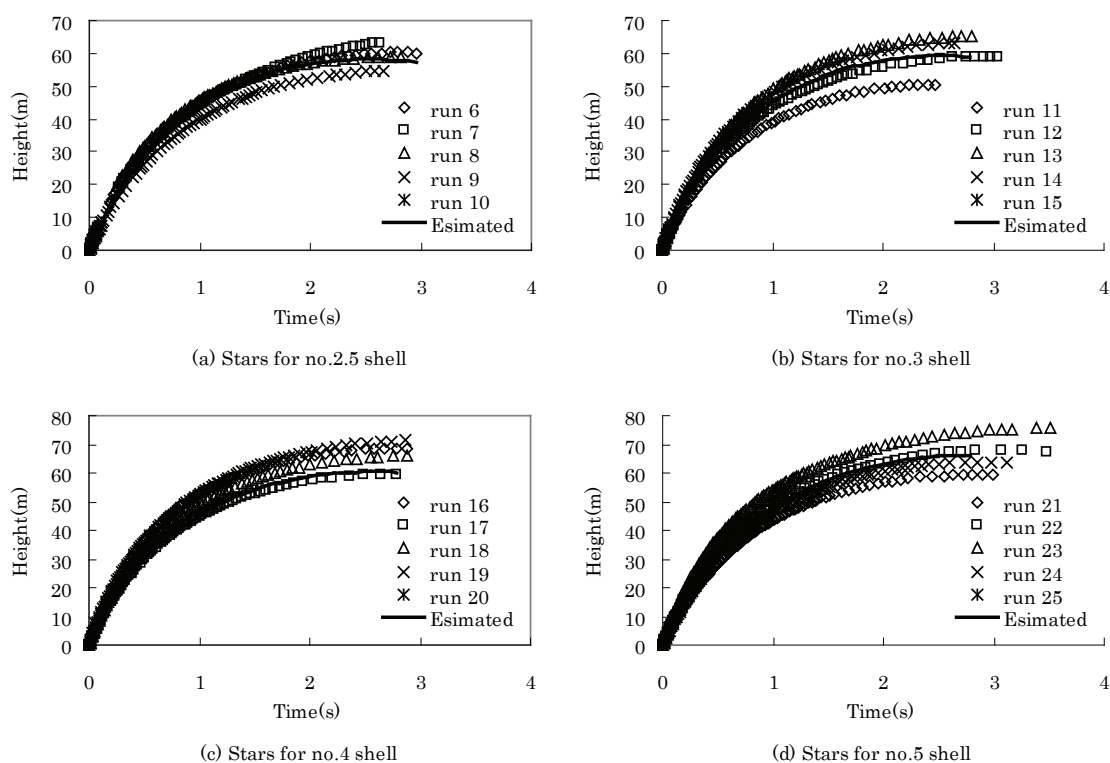


Figure 4 Estimated and observed trajectories of burning stars.

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