

# Prediction of Aerial Shell and Comet Trajectories Using SHELLCALC<sup>©</sup>

John Harradine<sup>a</sup> and Tom Smith<sup>b</sup>

<sup>a</sup>Manly, Queensland, Australia

<sup>b</sup>Davas Ltd, 8 Aragon Place, Kimbolton, Cambs PE28 0JD, UK

**Abstract:** *This paper describes a model for predicting the path of aerial shells and Roman candle comets. This model, incorporated in a Microsoft<sup>®</sup> Excel-based freeware program, SHELLCALC<sup>©</sup>, predicts the trajectory of these fireworks using point mass equations for range and height. These equations are modified to take into consideration mortar/candle angle, launch altitude above sea level, wind speed and direction, comet consumption, air density and terrain, and incorporate an approximation of shell drift through tumbling motion and mortar balloting. The graphical output from the model also incorporates typical shell burst diameters.*

**Keywords:** *Shells, comets, ballistics, trajectory, fall-out*

## Introduction

On many occasions it is useful to predict the path of an aerial shell or Roman candle effects (comets) given certain initial conditions. In response to this need, the authors have developed a Microsoft<sup>®</sup> Excel-based predictor, SHELLCALC<sup>©</sup>, which is easy to use, and provides graphical outputs which may be readily understood by the lay person and be included in reports and presentations.

SHELLCALC<sup>©</sup> may be used to assist:

- fireworks operators to plan a display, especially where a risk or hazard assessment is required;
- fireworks regulators to develop safe distances and establish risk regimes for fireworks displays;
- enforcement and investigative agencies to predict possible outcomes of “near misses”, or to confirm actual incident data, particularly for presentation in court or in investigation reports;
- fireworks testers to establish a safe template for test firings; and
- fireworks manufacturers to establish safe distances for their products and predict the effect of variable fuse delay times on burst height.

## General Layout

The SHELLCALC<sup>©</sup> data input and output screen is shown in Figure 1. All information is entered and displayed on the same page. In this way it is possible to readily examine the effect of changing a single parameter on the results calculated.

For instance it may be particularly useful to set the fuse delay time to the total flight time of the shell – to examine the possible spread of stars in this, one of the highest consequence failure modes of aerial shells.

## Program Input

SHELLCALC<sup>©</sup> requires that several parameters be input before a meaningful output can be achieved. These inputs, for the aerial shell and roman candle predictors, are shown in Figures 2 and 3, respectively. The user can choose between imperial or metric input, and between calculations on shells or comets. The difference choosing shells or comets is twofold – the graphical output only displays approximate shell burst sizes for shells, and the calculations for shells assume that the shell’s mass remains constant throughout its flight. Conversely for comets, no burst diameter information is displayed, and the calculations assume the comet’s mass decreases to zero during its flight time.

In addition to choosing the calibre of the shell or comet star, the user can also enter other optional parameters including the fuse delay time for shells

# SHELLCALC<sup>®</sup> v3.1

Developed by John Harradine, Manly, Queensland, Australia with additions by Tom Smith, Davas Ltd, UK - effective 1 October 2005

About SHELLCALC<sup>®</sup>

## Input

	Units	Metric
Type		Shells
Shell Diameter	6" (150mm)	
Mortar Angle		0 degrees from vertical
Muzzle Velocity		120 m/s
Fuse Delay		6 s
Shell Mass		g
Tumbling/Mortar Drift		Major <sup>n</sup>
Wind Speed		0 km/h
Relative Wind Direction		degrees (0 = tailwind, 180 = headwind, 90 = wind from right, -90 = wind from left)
Elevation of Launch Site		300 m AMSL
Terrain Category		2 (refer AS1170.2)

## Output

Max Downrange Carry	65 m
Max Height	228 m
Max Crossrange Carry	0 m
Approx Burst Diameter	150 m
Ascent Time	5.9 s
Flight Time	13.7 s
Shell Burst Height	227 m
Shell Mass	1217 g

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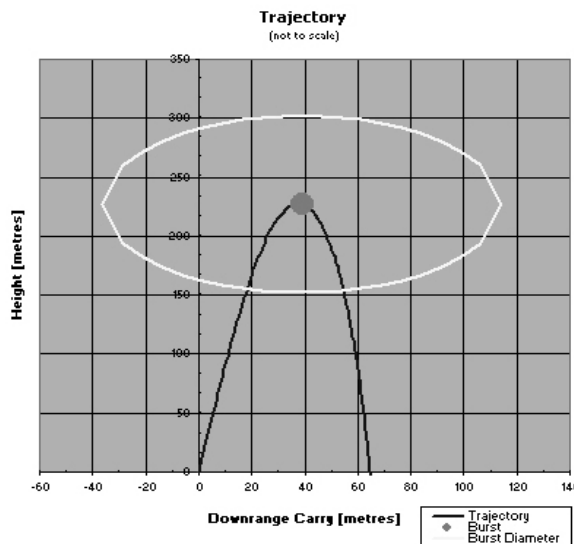
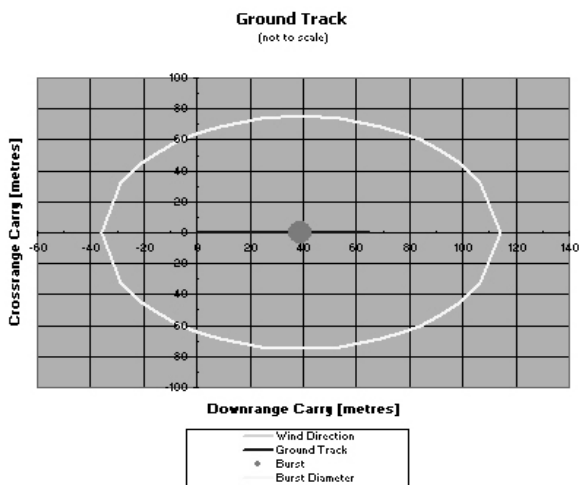


Figure 1. General layout of SHELLCALC<sup>®</sup> data input, data output and graphical output screen.

(if no value is entered the shell is assumed to burst at the apex of its flight), the mass of the shell (if no value is entered the mass is calculated), the muzzle velocity (default values are provided), the wind speed and direction, the elevation of the launch site and a parameter to reflect the terrain category.

## Trajectory Prediction

The basis of SHELLCALC<sup>®</sup>'s trajectory prediction is a point mass trajectory model which, in the case of the Roman candle predictor, has been modified

to take into account the burning of the comet during flight (described later). The point mass model is accepted as the simplest useful trajectory prediction technique which takes air resistance into consideration.<sup>1</sup> The point mass model predicts both range ( $x$ ,  $y$ ) and height ( $z$ ) components of acceleration for a projectile of given mass  $m$ . The Cartesian points of reference for the point mass model are shown in Figure 4.

## Input

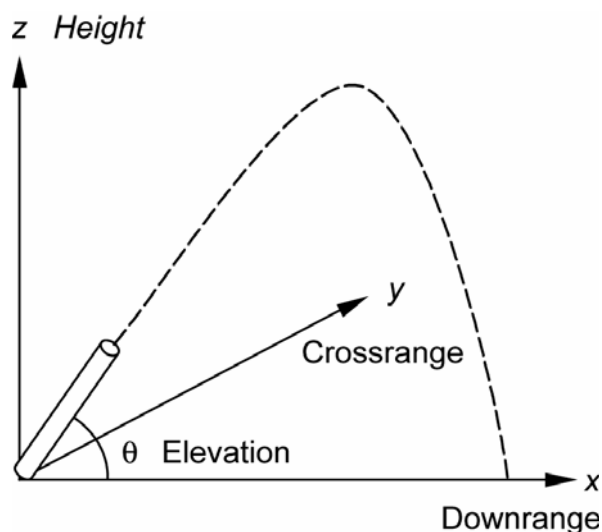
	Units	Metric
Type		Shells
Shell Diameter	6" (150mm)	
Mortar Angle		0 degrees from vertical
Muzzle Velocity		120 m/s
Fuse Delay		6 s
Shell Mass		g
Tumbling/Mortar Drift		Major <sup>n</sup>
Wind Speed		0 km/h
Relative Wind Direction		degrees (0 = tailwind, 180 = headwind, 90 = wind from right, -90 = wind from left)
Elevation of Launch Site		300 m AMSL
Terrain Category		2 (refer AS1170.2)

## Input

	Units	Metric
Type		Comets
Comet Diameter	6" (150mm)	
Roman Candle Angle		0 degrees from vertical
Muzzle Velocity		120 m/s
Comet Burning Time		6 s
Comet Mass		g
Tumbling/Mortar Drift		Major <sup>n</sup>
Wind Speed		0 km/h
Relative Wind Direction		degrees (0 = tailwind, 180 = headwind, 90 = wind from right, -90 = wind from left)
Elevation of Launch Site		300 m AMSL
Terrain Category		2 (refer AS1170.2)

Figure 2. Aerial Shell input parameters.

Figure 3. Comet Shell input parameters.



**Figure 4.** Cartesian space used in SHELLCALC<sup>®</sup>.

The equations for range and height are:

$$m\ddot{x} = \rho V^2 \pi d^2 C_D \left( \frac{\dot{x}}{8V} \right)$$

$$m\ddot{y} = \rho V^2 \pi d^2 C_D \left( \frac{\dot{y}}{8V} \right)$$

$$m\ddot{z} = -\rho V^2 \pi d^2 C_D \left( \frac{\dot{z}}{8V} \right) - mg$$

where:  $m$  = mass of projectile [kg]  
 $x$  = downrange distance [m]  
 $\rho$  = density of air [ $\text{kg m}^{-3}$ ]  
 $V$  = velocity of projectile [ $\text{m s}^{-1}$ ]  
 $d$  = effective diameter of projectile [m]  
 $C_D$  = drag coefficient of projectile [-]  
 $g$  = acceleration due to gravity [ $\text{m s}^{-2}$ ]  
 $y$  = crossrange distance [m]  
 $z$  = height [m]

These equations are numerically integrated to solve for  $x$ ,  $y$  and  $z$  based on a time interval of 0.1 s (this was found to give an acceptable level of accuracy without increasing the file size to an unacceptable level).

#### Correction for Drag Coefficient

The drag coefficient ( $C_D$ ) of aerial shells is, at best, difficult to quantify. Such factors as roundness,

surface roughness, effective Reynolds number and rotation will vary markedly from shell to shell and affect  $C_D$ , in some cases markedly. Given that these vagaries exist, an approximate value of  $C_D$  for shell diameters of 2 in (50 mm) to 12 in (300 mm) was determined by applying a best-fit model to Shimizu's empirical work on aerial shells,<sup>2</sup> resulting in:

$$C_D = -0.0921 h D + 0.9283$$

where  $D$  = effective presented diameter of projectile to airflow [mm]

Shimizu's  $C_D$  values are most likely on the low side; he used Japanese shells which were most likely of better quality than those of Chinese manufacture, which represent the majority of shells used worldwide today.

$C_D$  for comets is more difficult to predict, as a comet in flight is a deflagrating cylinder that is inherently unstable. For simplicity's sake, the  $C_D$  of comets was taken to approximate that of an aerial shell of similar diameter.

#### Estimated Shell Mass

An approximate value of mass,  $m$ , for aerial shells with diameters of 2 in (50 mm) to 12 in (300 mm) was determined by applying best fit to values from various sources:<sup>2,3</sup>

$$m = 205.25d^{2.7029}$$

The user can enter the mass of the aerial shell in question if this is known; otherwise the program will use the estimated value.

#### Correction for Comet Mass Consumption

Unlike aerial shells, which are assumed to remain intact during flight, an allowance must be made for the consumption of the burning comet in flight. If the effect of pressure on burning rate is ignored (that is, burning rate remains constant), and surface burning of a cylindrical comet is assumed, the mass of the comet  $m$  after elapsed time  $t$  can be calculated using:

$$m = \frac{\pi \rho_p}{4} (d_0 - 2rt)^2 (l_0 - 2rt)$$

**Table 1.** Typical dimensional and burning rate parameters for a 2 in (50 mm) roman candle comet.<sup>4</sup>

Parameter	Value
$m$	0.068 kg
$\rho_p$	1860 kg m <sup>-3</sup>
$d_0$	0.045 m
$l_0$	0.023 m
$r$	0.00143 m s <sup>-1</sup>

where:  $\rho_p$  = density of comet [kg m<sup>-3</sup>]  
 $d_0$  = initial diameter of comet [m]  
 $l_0$  = initial thickness of comet [m]  
 $r$  = linear burning rate of comet composition [m s<sup>-1</sup>]

Typical values for these parameters for 2 in (50 mm) roman candles are shown in Table 1. These values for density  $\rho_p$  and burning rate  $r$  should be reasonably accurate when applied to all solid black-powder or flash powder effects (including comets and stars), and these values are used by default in SHELLCALC<sup>®</sup>. The user has the option of inputting a different comet burn time if this is known; in this instance, the program will alter  $r$  to achieve the required total burn time.

### Effective Comet Diameter

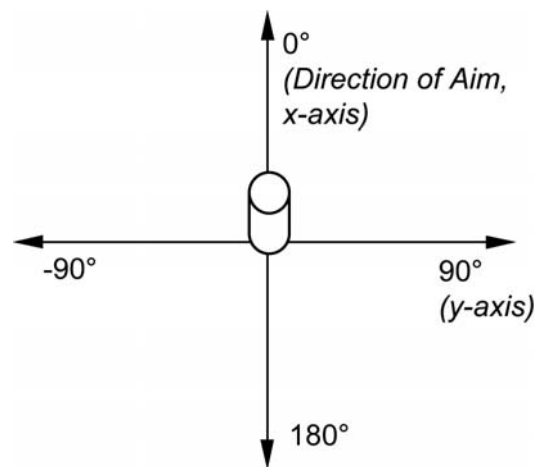
As a comet is consumed during flight, its effective diameter reduces, thereby reducing the effective presented area of the comet. SHELLCALC<sup>®</sup> uses the following formula to achieve an approximation of the effective presented diameter,  $d$ :

$$d = d_0 - 2rt$$

### Correction for Wind

SHELLCALC<sup>®</sup> permits the user to input wind direction and speed relative to the direction of aim of the mortar or Roman candle. The program requires the user to input a wind bearing relative to the direction of aim as shown in Figure 4. For example, a headwind has a relative bearing of 0°, and a wind from the right has a bearing of 90°.

Based on the wind direction and speed input by the user, SHELLCALC<sup>®</sup> resolves the velocity vectors in the  $x$  and  $y$  axes and adjusts the values of  $V$ ,  $\dot{x}$  and  $\dot{y}$  in the relevant point mass equations.



**Figure 5.** Relative wind bearings used in SHELLCALC<sup>®</sup>.

### Correction for Mortar/Roman Candle Angle

When the mortar or roman candle is set at an angle other than vertical, the user can nominate the appropriate angle from vertical. Based on this angle, SHELLCALC<sup>®</sup> calculates the mortar elevation  $\theta$  (refer Figure 1), resolves the velocity vectors in the  $x$  and  $z$  axes and adjusts the values of  $\dot{x}$  and  $\dot{z}$  in the relevant point mass equations.

### Correction for Terrain

Terrain affects actual wind speed at ground level. If turbulence and other anomalies are ignored, a good estimate of wind speed  $w$  for a given height above ground level  $z$  can be estimated if the AS/NZS 1170.2 terrain category is known.<sup>5</sup> The value of  $w$  for terrain category 2 (open spaces and water) is:

$$w = w_0(0.1036 \ln(z+1) + 0.8731)$$

where  $w_0$  = wind speed out of ground effect [ms<sup>-1</sup>].

Similarly, the value of  $w$  for terrain category 3 (sports grounds and built-up areas) is:

$$w = w_0(0.139 \ln(z+1) + 0.7503)$$

Based on user input, SHELLCALC<sup>®</sup> corrects  $V$  using the appropriate equation. If the user does not nominate a terrain category, the program ignores terrain effects on  $V$ .

## Correction for Air Density

As air density  $\rho$  changes with altitude, SHELLCALC<sup>®</sup> calculates  $\rho$  at each 0.1 second increment and uses this value in the relevant point mass equations. Air density at a given altitude ( $z + z_0$ ) is calculated according to the following equation:

$$\rho = \rho_0 e^{-0.0001065(z+z_0)}$$

where:  $\rho_0$  = density of air at sea level [ $\text{kg m}^{-3}$ ]  
 $z_0$  = elevation of launch site above sea level [m]

## Program Output

Once the user inputs all data, SHELLCALC<sup>®</sup> will provide both numerical (Figure 6) and graphical (Figures 7, 8 and 9) outputs. The graphical outputs are Excel charts, and so may be copied to other applications for inclusion in reports or presentations.

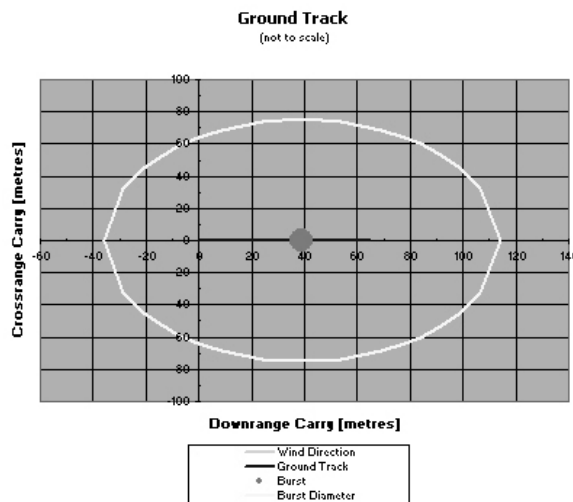
### Shell Burst Diameters

Shellcalc<sup>®</sup> also displays the typical burst diameter of shells at either the time selected by the user, or if this parameter is not entered, at the apex of the shell's flight. The values for typical shell burst diameters are taken from work on Japanese Shell Break Radii<sup>7</sup> and are a mathematical close fit to the typical values cited in that paper (Figure 10). However, the authors noted that the maximum observed break radii can exceed the typical values by a significant amount, and that shells from different suppliers, and shells comprising different effects may also deviate significantly from the "typical" values. The user is cautioned to consider the graphical output as illustrative only.

### Output

Max Downrange Carry	38 m
Max Height	236 m
Max Crossrange Carry	0 m
Approx Burst Diameter	150 m
Time at All Burnt	6.0 s
Height at All Burnt	235 m
Comet Mass	1217 g

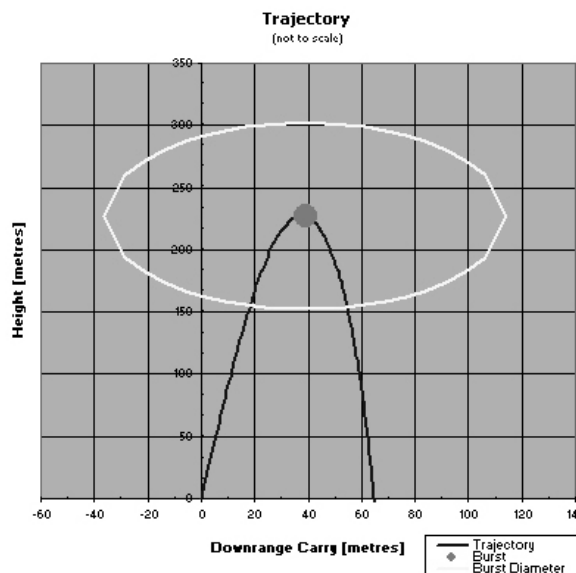
**Figure 6.** Numerical output for aerial shell prediction.



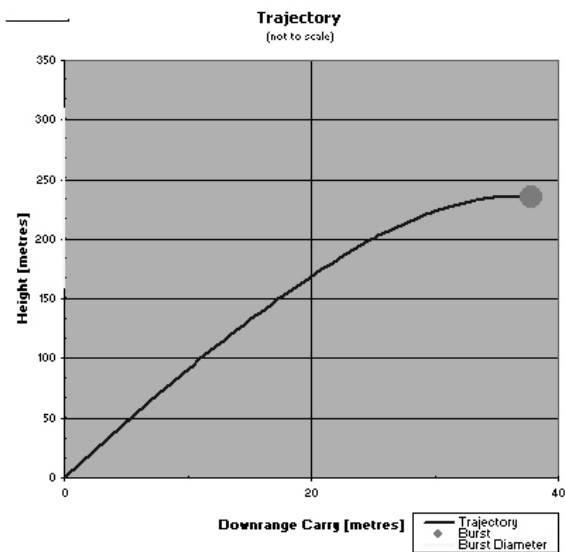
**Figure 7.** Ground track of aerial shell showing location of shell burst and approximate shell burst size

### Shell Drift

Firework shells are subject to several mechanisms to account for the observed shell drift, even in completely still conditions. Such factors include shell spin (Magnus effect) and shell balloting due to the fit of the shell in a mortar and the mortar length. SHELLCALC<sup>®</sup> uses shell balloting data from Norton<sup>7</sup> to attempt to factor in, albeit in a crude way, the observed deviations. SHELLCALC<sup>®</sup> allows the user to select three values related to this effect,



**Figure 8.** Trajectory of aerial shell showing location and approximate shell burst size.



**Figure 9.** Trajectory of comet showing location of “burn out”.

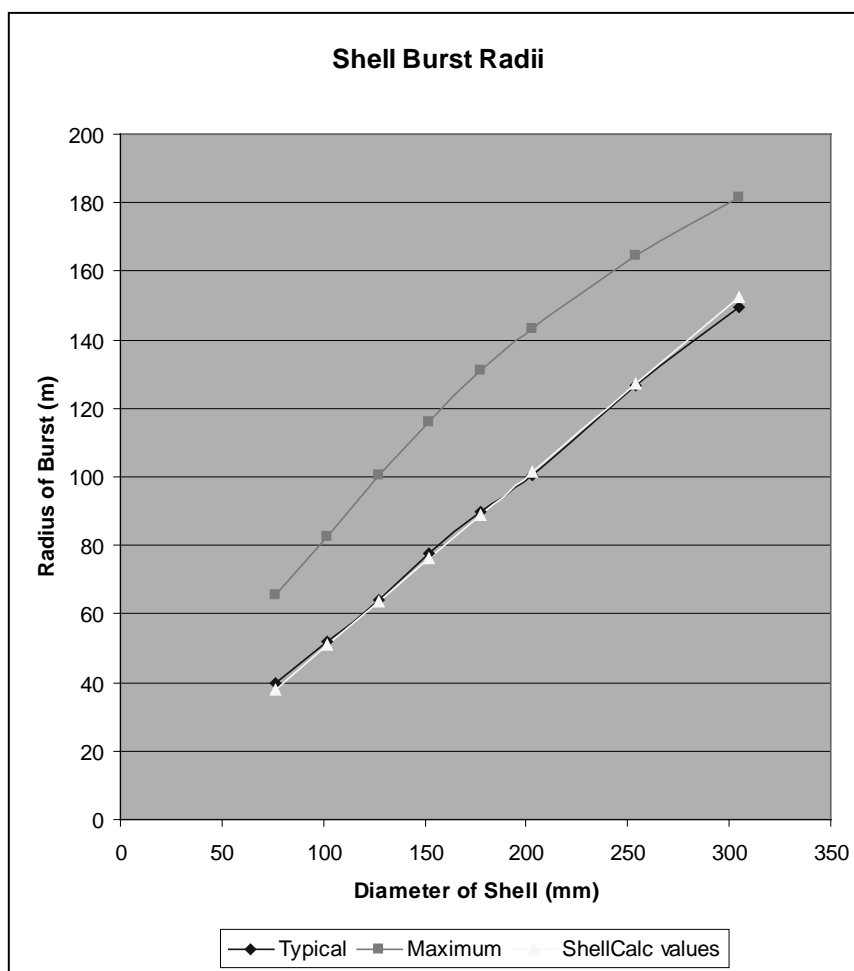
and to incorporate this into the final output values and graphs.

**None** – shell drift parameters are ignored

**Typical** – an average value of shell deviation from Norton is used (approx. 2° deviation)

**Maximum** – the largest value of shell deviation calculated by Norton is used (approx. 5° deviation)

The user should note that shell drifts are due to many factors, and the incorporation of this parameter into SHELLCALC<sup>©</sup> is largely to prevent the program calculating that in still conditions, where a shell is fired vertically, that it would necessarily return to earth exactly at the point of firing. It is also useful to use this parameter when estimating “worst case” scenarios, for instance angled mortars, significant tailwind, maximum shell drift and



**Figure 10.** Typical, Maximum and SHELLCALC<sup>©</sup> values for burst radius as function of shell diameter used in SHELLCALC<sup>©</sup> calculations.

shell bursting at ground level. The extent of shell balloting depends on the shell–mortar clearance and to the effective length of the mortar and in general better shell–mortar fit and longer mortars decrease this effect.

### **Agreement with Observation**

Based on the authors' observation and Shimizu's work,<sup>2</sup> the predictions made by SHELLCALC<sup>®</sup> are close to the actual behaviour of aerial shells and Roman candle comets. As SHELLCALC<sup>®</sup> is still a basic program with limited accuracy, the authors would appreciate any feedback on improvements.

### **References**

- 1 Aeromechanical Systems Group, *Basic External Ballistics Lecture Notes*, Royal Military College of Science, Shrivenham, UK, 1992.
- 2 T. Shimizu (translated by A. Schuman), *Fireworks From a Physical Standpoint – Part III*, Pyrotechnica Publications, 1985.
- 3 J. B. Harradine, *Chemical and Physical Analysis of Roman Candles, Other Types of Fireworks and Their Residues*, Explosives Inspectorate, Department of Natural Resources and Mines, Brisbane, 2001.
- 4 J. B. Harradine, *Dynamic Testing of Roman Candles and Other Types of Fireworks, Safety and Health Division*, Department of Natural Resources and Mines, Brisbane, 2001.
- 5 Standards Australia, *AS/NZS 1170.2:2002: Structural design actions - Wind actions*, 2002.
- 6 K. L. and B. J. Kosanke, *Pyrotechnics Guild International Bulletin*, No. 59 (1988), p. 104.
- 7 Randall K. Norton, *Journal of Pyrotechnics*, Issue 13, 2001, p. 31.

### **Note**

SHELLCALC<sup>®</sup> is a freeware program and is designed to run on Microsoft<sup>®</sup> Excel 95 or later. Copies and updates may be obtained directly from the download section of the Journal of Pyrotechnics Website:

<http://www.jpyro.com/downloads/shellcalc/>