# Interior Pressure in the Mortar and Motion of a No. 3 Shell in a Fireworks Shot 

Yuzo Ooki, Dayu Ding, Masamori Higaki and Tadao Yoshida*<br>Department of Mechanical Engineering, Ashikaga Institute of Technology<br>268-1 Omae-cho, Ashikaga-shi, Tochigi 326-8558, Japan<br>*To whom all correspondence should be addressed email: yoshida@ashitech.ac.jp


#### Abstract

Interior pressure in a fireworks mortar and initial velocity for the shot of a no. 3 shell were measured and results were analyzed. The observed initial velocity was mainly explained by the action of the interior pressure during the shot. On the other hand, a delay in pressure propagation in the mortar, the effect of gas flow through the gap between the mortar and shell, and acceleration of the shell just after leaving the muzzle were suggested.


Keywords: fireworks, shot, interior ballistics, initial velocity

## Introduction

Spherical shells are typical Japanese fireworks, shot from a mortar, developing in the sky releasing burning stars, in the form of various artificial flowers. These phenomena can be divided into the behavior of the shell in the mortar and in the air, the bursting of the shell, and the motion of the burning stars. In this article, we describe the results of a study on only the interior ballistics, and the relationship between pressure profile in the mortar and initial shell velocity using a no. 3 shell.

There is a pioneering work on the interior ballistics of the shot of a spherical firework shell by Shimizu. ${ }^{1}$ His work was based on the interior ballistics of guns and considered the special situation of a firework shell and mortar. His work used a sophisticated theory to understand the situation and modern workers find it hard to use.

The initial velocity of model shells has been measured using a high-speed video camera by Matsunaga, Yoshida et al, ${ }^{2}$ and the pressure profile in the mortar has been recorded by Matsunaga et al. ${ }^{3}$ However the relationship between the pressure profile and the initial velocity has not so far been analyzed and reported.

We carried out the shot experiments using a pressure sensor, amplifier, digital oscilloscope and high-speed video camera, and results were analyzed using a personal computer. In our
present work, the interior pressure of the mortar for a no. 3 shell and the trajectory of the shell in the air were recorded, and the results are analyzed and discussed.

## Experimental

## Materials

No. 3 firework shells, lifting charges and electric matches were provided by Sunaga Fireworks Co. Ltd. The lifting charge was grain black powder made by Nippon Kayaku Co. Ltd.

## Apparatus

The fireworks mortar is shown in Fig. 1. The inner diameter, wall thickness, depth and total height of the mortar were $0.09 \mathrm{~m}, 0.006 \mathrm{~m}, 0.75 \mathrm{~m}$ and 0.78 m , respectively. Two pressure sensors were fitted, to the bottom and to a position 0.26 m below the muzzle of the mortar.

The pressure in the mortar during the shot was measured using two pressure sensors (Kistler 60410A), charge amplifiers (Kistler 5011) and a digital oscilloscope (Sony Tektronix TDS3012). Two pressure sensors were fitted at the bottom and middle of the mortar. The front surface of the sensors was covered with grease to protect the surface from the heat of combustion of the gas produced by the lifting charge.The initial velocity


Figure 1. Fireworks mortar and pressure measuring system.
of the shell was measured using a high-speed video camera (FOR.A VFC-100SB).

## Procedure

The mortar was set on the ground vertically. The lifting charge and electric match in a polyethylene bag were put in the bottom of the mortar. Then a no. 3 shell was placed on the lifting charge. The electric match was ignited by turning on an electric current. The lifting charge burned, pressure developed and the shell moved upwards. The pressure profile was recorded on an oscilloscope and the initial trajectory of the shell in the air was recorded on a high-speed camera. Each frame of the video was reproduced on a video screen and the initial velocity of the shell was determined.

## Results and Discussion

## Pressure Profile

Pressure profiles in the mortar during shot of the shell are shown in Fig. 2. The pressure profile shown by the thick line was recorded by the bottom sensor, and the profile shown by the fine line by the middle sensor. Both profiles in a figure were recorded simultaneously in a shot.

The profile from the bottom sensor showed an Sshaped curve in the first stage (from the middle pressure sensor to the bottom pressure sensor) and a sharply decreasing curve in the last stage (from the bottom pressure sensor). The first stage corresponds to the period of time during which the


Figure 2(a)-(h). Pressure profiles in the mortar.

## (i)



Figure 2 cont. (i). Pressure profiles in the mortar.
shell is traveling in the mortar. The later stage corresponds to the period after the shell has left from the muzzle.

The profile from the middle sensor shows a pressure drop just before the sharp pressure increase, and after the sharp pressure increase the pressure profile is similar to that of the bottom sensor. The pressure drop before the sharp increase may be caused by the rapid flow of the combustion gas through the gap between the shell and the mortar wall. This phenomenon may cause the shell to accelerate.

The pressure recorded by the middle sensor is lower than that recorded by the bottom sensor suggesting a delay in pressure propagation in the mortar. At the moment, this is merely our speculation.

## Motion of a Shell in the Mortar

The motion of a shell in the mortar was analyzed using a personal computer. The equations of motion of a shell are expressed as follows:
$M \frac{\mathrm{~d} u}{\mathrm{~d} t}=p(t) \times A-M g$
$\frac{\mathrm{d} Z}{\mathrm{~d} t}=u$
Here, $M, u, A$, and $Z$ are mass, motion velocity, maximum cross sectional area, and traveling distance of the shell, respectively.
$A=\frac{\pi D^{2}}{4}$

Here, $D$ is the diameter of the shell, and

$$
\begin{align*}
\frac{\mathrm{d} u}{\mathrm{~d} t} & =\frac{\pi D^{2}}{4 M} \cdot p(t)-g  \tag{4}\\
\frac{\mathrm{~d} Z}{\mathrm{~d} t} & =u \tag{5}
\end{align*}
$$

Here, $p(t)$ is observed value and substituted into Equation (4).

Equations (4) and (5) are simultaneously solved by numerical calculation, and acceleration $\mathrm{d} u / \mathrm{d} t$, velocity $u$ and traveling distance $Z$ are obtained.

In practice, $g$ is much less than
$\frac{\pi D^{2}}{4 M} \cdot p(t)$
and can be neglected.
Equations (4) and (5) were solved by the RungeKutta method. The time integration process for ordinary differential equations (4) and (5) was performed using a fourth order accuracy RungeKutta method.

The digital pressure data were recorded on an oscilloscope and the data were reduced using Excel. These reduced data were used for calculating acceleration, velocity and traveling distance of the shell.

An example of the calculated profiles of the acceleration is shown in Fig. 3(a). A velocity profile is shown in Fig. 3(b), and a distance profile in Fig. 3(c).

The weight of the lifting charge, calculated maximum acceleration, velocity and distance of the shell in the mortar, and observed initial velocity are listed in Table 1.

## Observed initial velocity of the shell in the air and calculated muzzle velocity

When a shell is shot, smoke and flame appear from the muzzle and then the shell appears above the smoke. We cannot observe real muzzle velocity. We can only determine initial velocity after the shell appears from the smoke. In the same experiment with the pressure measurement, the initial velocity
(a)

(b)

(c)


Figure 3(a)-(c). Profiles of calculated acceleration (a), velocity (b), and traveling distance (c) of a shell for shot 1 .
of the shell was determined. The shell appeared above the smoke at about 2 m above the muzzle and the time difference to about 5 m high was determined. Then the initial velocity was derived. The observed initial velocities were similar to the calculated muzzle velocities from the pressure profiles in the mortar as listed in Table 1.

## Effect of residual pressure after a shell has left the muzzle

Pressures at the bottom and middle of the mortar drop sharply after a shell has left the muzzle, but positive pressures remain for a short time. This short-lived pressure accelerates the shell after leaving the muzzle and the velocity of the shell increases in calculation. But this is not realistic and should be made clear in the future.


Figure 4. Plot of observed initial velocity of shell vs. weight of lifting charge.


Figure 5. Correlation of observed initial velocity and muzzle velocity from pressure profile.

## Effect of weight of lifting charge on initial velocity of shell

Initial velocity of the shell is plotted against weight of lifting charge in Figure 4. Correlation between initial velocity and weight of lifting charge is not good, suggesting that the burning of lifting charge is irregular. This must be also proved by suitable experiment.

## Correlation of observed initial velocity of shell with calculated muzzle velocity from pressure profile

Calculated muzzle velocity from pressure profile is plotted against observed initial velocity by a highspeed video camera in Figure 5. The correlation is much better than that between observed initial velocity and weight of lifting charge.

Table 1. Weight of lifting charge, calculated and observed initial velocity, and calculated distance (A to B)

| Test no. | Mass of lifting <br> charge $/ \mathrm{g}$ | Calculated muzzle <br> velocity $/ \mathrm{m} \mathrm{s}^{-1}$ | Observed initial <br> velocity $/ \mathrm{m} \mathrm{s}^{-1}$ | Calculated distance $/ \mathrm{m}$ |
| :---: | :---: | :---: | :---: | :---: |
| Shot 1 | 30 | 141 | 135 | 0.69 |
| Shot 2 | 28 | 128 | 122 | 0.69 |
| Shot 3 | 24 | 137 | 132 | 0.69 |
| Shot 4 | 22 | 81 | 82 | 0.69 |
| Shot 5 | 20 | 76 | 78 | 0.69 |
| Shot 6 | 18 | 92 | 89 | 0.70 |
| Shot 7 | 16 | 87 | 83 | 0.70 |
| Shot 8 | 14 | 90 | 88 | 0.69 |
| Shot 9 | 12 | 72 | 70 | 0.70 |

Practically, the initial velocity of a firework shell can be estimated from the pressure profile, but not by the weight of the lifting charge at the moment.

## Calculated maximum distance of shell in the mortar

The real distance of a no. 3 shell from the bottom to the muzzle of the mortar is 0.69 m . Calculated maximum distances of the shell are listed in Table 1. Agreement of both distances is good.

However we should examine whether the observed pressure profile is correct or not. We should examine the effects of delay of pressure propagation and of gas flow through the gap between the shell and the mortar wall in the future.

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