Choked Flow, a Frequently Misunderstood Term

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ABSTRACT

A brief description of compressible fluid flow is presented to clarify and, hopefully, reduce the incorrect usage of the term "choked flow" in the fireworks community.

Keywords: choked flow, fluid flow, gas velocity, mass flow, rocket, sonic flow

A term that is frequently used in the field of fluid mechanics is "choked flow". Unfortunately, it is also a frequently misused term. The author has found that this seems to be particularly true in the fireworks industry where it has been used as an explanation for, among other things, the transition between the initial slow buildup of pressure in fireworks mortars and a sudden and rapid increase in mortar pressure. (Choked flow has also been similarly invoked as one explanation of exploding gerbs.) This article is a brief explanation of those conditions under which the use of the term choked flow would be correct, and why and under what conditions, especially in fireworks, its use is incorrect.

Figure 1 presents two examples of gas flow from some pressure source region labeled P_0 and having a pressure of P_0 through a constricted throat section similarly labeled P_1 (at pressure P_1) to the atmosphere labeled P_2 (at ambient pressure P_2). The upper depiction in Figure 1 is typical of either a rocket motor or gerb, and the lower depiction shows the somewhat analogous situation of a spherical shell firing from a mortar. If the pressures P_0 and P_2 are equal, there will be no flow of gas. If the pressure P_0 is raised above P_2, the gas will begin to flow with some velocity, with the point of maximum constriction at P_1 being of particular interest. If P_0 is increased further, the velocity of the flow at point P_1 again increases. However, if the pressure P_0 continues to be increased, at some point the velocity of the gas flow at point P_1 will reach the speed of sound (which for a given gas is mostly a function of temperature). At that point, any further increase in P_0 will not result in a further increase in gas flow velocity at point P_1. This is the condition generally described as "choked flow".

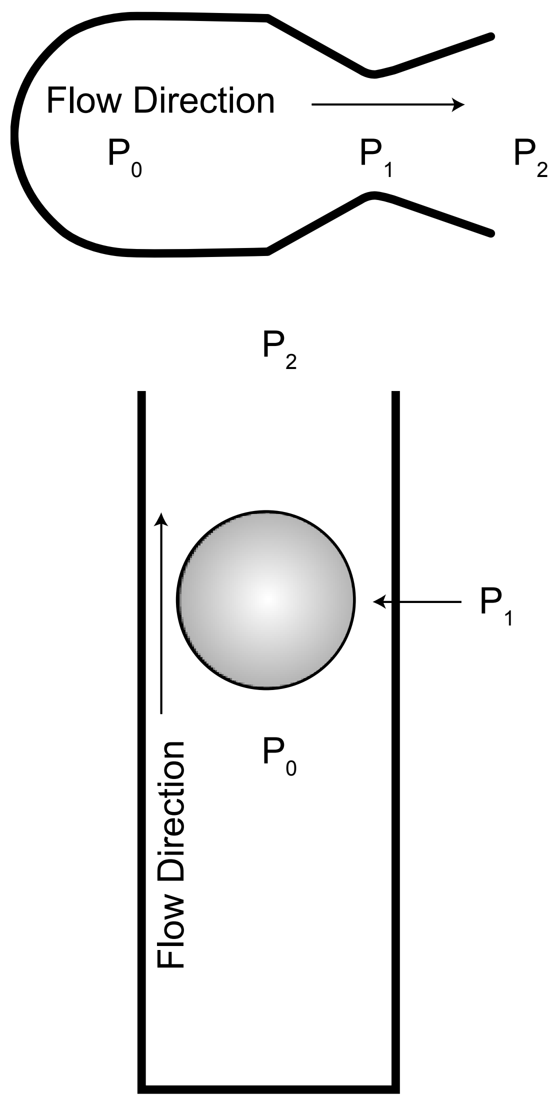

Figure 1. Examples of gas flow in items such as a rocket motor or gerb (above) and a discharging fireworks mortar (below).

Figure 2 is a somewhat typical graph of mortar pressure as a function of time during the firing of an aerial shell. Note that there is a rather long interval between igniting the lift charge with an electric match (at t_0) and the eventual rapid rise in mortar pressure (occurring from t_r to t_p). It has occasionally been suggested that the reason for the sudden onset and rapid increase in pressure was that the velocity of the gas escaping around the aerial shell has reached the speed of sound. Since there can be no further increases in the velocity of the escaping lift gas as mortar pressure continues to increase, it is suggested that this results in something like a piling up of gas that is unable to escape, and this is what causes the precipitous rise in mortar pressure. However, as is demonstrated below, an examination of gas flow dynamics finds that this argument cannot be supported.

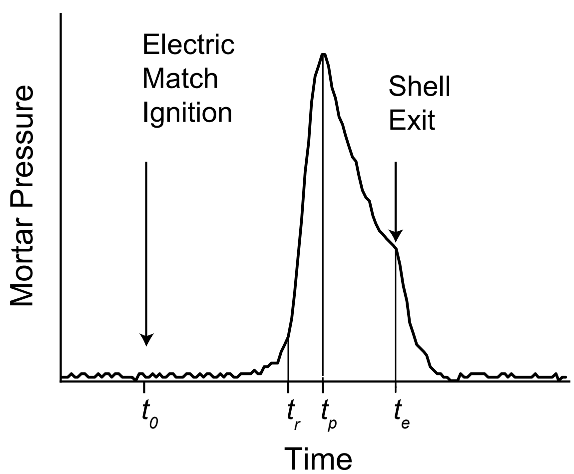

Figure 2. A graph of mortar pressure (gauge pressure) as a function of time during the firing of an aerial shell.

Before beginning the discussion of gas dynamics in general, and choked flow in particular, it is appropriate to point out that the information presented in this article can be found in any number of text books. For this reason, specific references are not included, but rather a list of general references is provided at the conclusion of the article.

The first step in discussing choked flow is to demonstrate that under the conditions assumed for this article, the density of a gas is proportional to its pressure. Equation 1 is known as the Ideal Gas Law and is a reasonably accurate equation of state for most commonly encountered gases under the conditions of pressure and temperature encountered in fireworks and rocketry.

$$PV = nRT \qquad (1)$$

where P is absolute pressure (as opposed to gauge pressure, or pressure above atmospheric), V is volume, n is the number of moles of gas, R is a constant of proportionality (the Universal Gas Constant, the magnitude of which depends on the system of units being used), and T is the absolute temperature.

In this discussion, only pressure sources venting to the ambient atmosphere will be considered, and the temperature (T) at the pressure source will be considered to be constant. Therefore, eq 1 reduces to

$$PV \propto n \quad or \quad P \propto \frac{n}{V} \qquad (2)$$

(Equation 2 is also known as Boyles Law.) Since density (ρ) is defined as mass divided by volume, and number of moles of a gas is proportional to the mass of that gas (m), then

$$\rho = \frac{m}{V} \propto \frac{n}{V} \qquad (3)$$

For eqs 2 and 3 both to be true, for an ideal gas its density must be directly proportional to the pressure, ($\rho \propto P$) (i.e., gas density increases linearly with gas pressure).

The second step in this discussion is to derive a general equation for the mass flow rate for a gas in motion. Consider a gas flowing through a pipe, such as illustrated in Figure 3, with a known constant velocity (v). During a given time interval (t), not considering the random motions of the individual gas molecules, the gas starting at point 1 will have traveled to point 2. In this case, the distance traveled (D) will equal gas velocity times time.

$$D = v \cdot t \qquad (4)$$

During that same time interval, the volume of the gas (V) passing point 1 will be that amount of gas in the volume of the pipe between points 1 and 2, which is equal to the cross sectional area of the pipe (A) times the distance between points 1 and 2 (D).

$$V = A \cdot D = A \cdot v \cdot t \tag{5}$$

Since density is defined as equaling mass (m) divided by volume (V), the mass of gas passing point 1 during this same time interval is equal to the density of the gas times its volume.

$$m = \rho \cdot V = \rho \cdot A \cdot v \cdot t \tag{6}$$

Mass flow rate ($\dot{m}$) is defined as the mass passing point 1 divided by the time that has elapsed.

$$\dot{m} = \rho \cdot A \cdot v \tag{7}$$

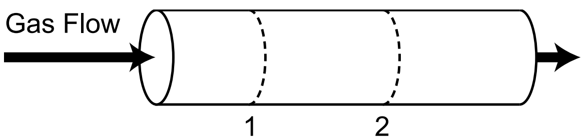

Figure 3. An illustration of a gas flowing through a simple pipe.

In the discussion of choked flow, it is the mass flow rate ($\dot{m}$) through the point of constriction that will be of primary interest. Also, when the terms "sub-sonic", "sonic", and "super-sonic" flow are used, they refer to the local speed of sound in the gas at the section being referred to (i.e., in sections P_0, P_1, or P_2). They do not refer to the speed of sound in the surrounding atmosphere. Finally, it should be noted that the local speed of sound is, for a given fluid (gas), primarily, but not exclusively, dependent on the temperature of the fluid. In the cases being discussed here, the actual value for the local speed of sound will not be specified, and it will be assumed to be a constant through out the device, as will the temperature and the chemical and molecular composition of the gas.

The basic equation for sub-sonic mass flow rate (eq 8) has been taken from standard reference texts and is presented here without derivation (for more information, see the list of references at the end of this article).

$$\dot{m} = A\sqrt{2\rho P_0} \times \left(\sqrt{\frac{k}{k-1}}\right) \times \sqrt{\left(\frac{P_2}{P_0}\right)^{\frac{2}{k}} - \left(\frac{P_2}{P_0}\right)^{\frac{k+1}{k}}} \tag{8}$$

In eq 8, P_0 is the chamber pressure in section P_0, P_2 is the ambient pressure in section P_2, A is the cross sectional area of section P_1, ρ is the density of the gas, and k is the ratio of the specific heat at constant pressure divided by the specific heat at constant volume (i.e., $k = C_p / C_v$) for the gas in the system. For common atmospheric gases, k is approximately 1.4.

Assume for the purposes of discussion that density is constant (and not proportional to pressure as was shown above). Then, if k is assumed to be 1.4, the pressure in section P_2 is held constant at atmospheric pressure and the pressure in section P_0 is increased from atmospheric to that which produces sonic flow in section P_1, eq 8 can be used to calculate mass flow rate. In this case, Figure 4 is a graph of the resulting mass flow rate, normalized to that when P_0 is 1.89 atmospheres.

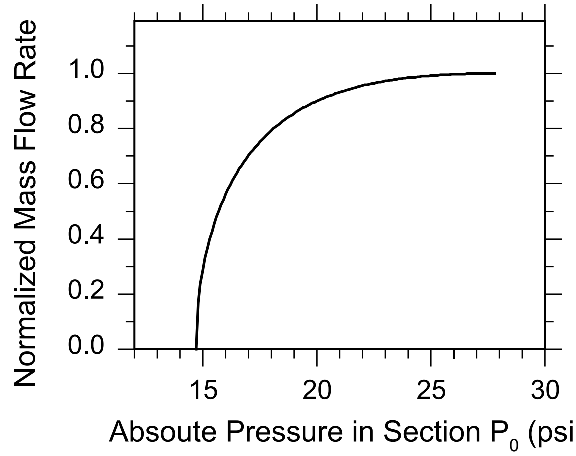

Figure 4. A graph of normalized sub-sonic mass flow rate as a function of absolute chamber pressure, incorrectly assuming gas density is constant, independent of pressure.

This type of curve can give rise to the term "choked flow", as it appears that no matter how high the pressure rises, the mass flow rate reaches a limit. (This is actually the case when the pressure, in the source chamber, P_0 is held constant and the exhaust pressure P_2 is reduced, such as would happen if a rocket were to travel into space.). However, recall that the graph in Figure 4 was based on the incorrect assumption that gas density in section P_2 was constant and not proportional to pressure (P_2).

Therefore, a term needs to be introduced in the first radical that will cause the density of the gas to be proportional to the pressure above ambient. Notice in eq 9, that when the pressure P_0 is the same as the ambient pressure, P_2, the term is equal to 1, and if the source pressure is twice ambient, the term is equal to 2, and so forth. (In eq 9, ρ_2 is the density of the gas at atmospheric pressure, P_2.)

$$\dot{m} = A\sqrt{2\rho_2\left(\frac{P_0}{P_2}\right)P_0} \times \left(\sqrt{\frac{k}{k-1}}\right) \times \sqrt{\left(\frac{P_2}{P_0}\right)^{\frac{2}{k}} - \left(\frac{P_2}{P_0}\right)^{\frac{k+1}{k}}} \qquad (9)$$

Equation 9, when plotted similarly to eq 8, results in the graph in Figure 5.

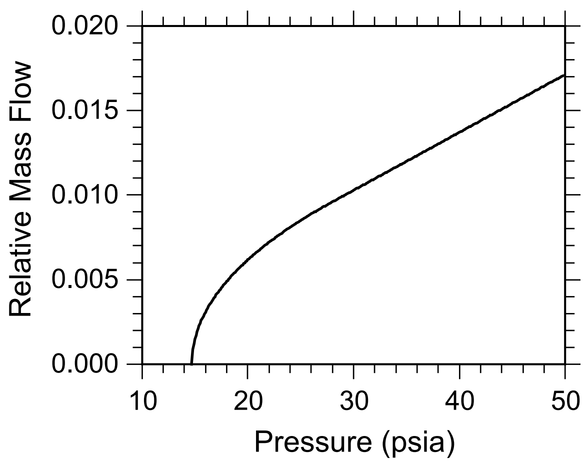

Figure 5. A graph of normalized sub-sonic mass flow rate as a function of absolute chamber pressure, correctly considering gas density to be proportional to pressure.

Equation 9 holds true so long as the flow remains sub-sonic (i.e., until the 'critical' pressure is reached). This critical pressure is defined as when the ratio of the pressure P_0 divided by P_2 exceeds the number given by

$$\left(\frac{k+1}{2}\right)^{\frac{k}{k-1}} \qquad (10)$$

At this critical pressure, the velocity in the 'throat' section P_1, is at sonic velocity, and for all higher source pressures (P_0) the following equation governs:

$$\dot{m} = A\sqrt{k\rho P_0} \times \left(\frac{2}{k+1}\right)^{\frac{k+1}{2k-2}} \qquad (11)$$

And, similarly to eq 9, a term is added to account for gas density in the chamber region being proportional to chamber pressure, giving

$$\dot{m} = A\sqrt{k\rho_2\left(\frac{P_0}{P_2}\right)P_0} \times \left(\frac{2}{k+1}\right)^{\frac{k+1}{2k-2}} \qquad (12)$$

A plot of eq 12 results in the graph shown in Figure 6.

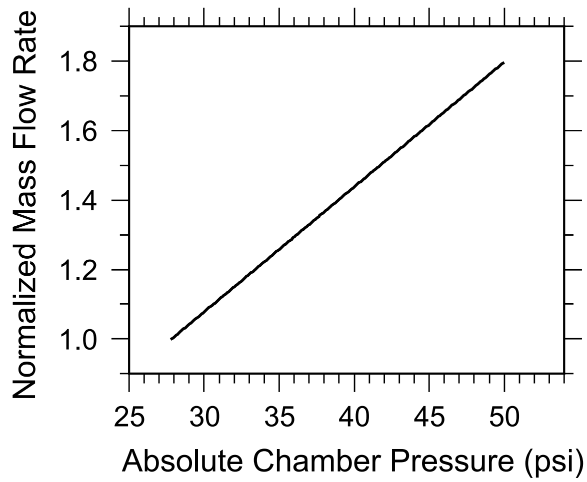

Figure 6. A graph of normalized super-sonic mass flow rate as a function of absolute chamber pressure.

Combining the sub-sonic and super-sonic data results in the graph shown in Figure 7.

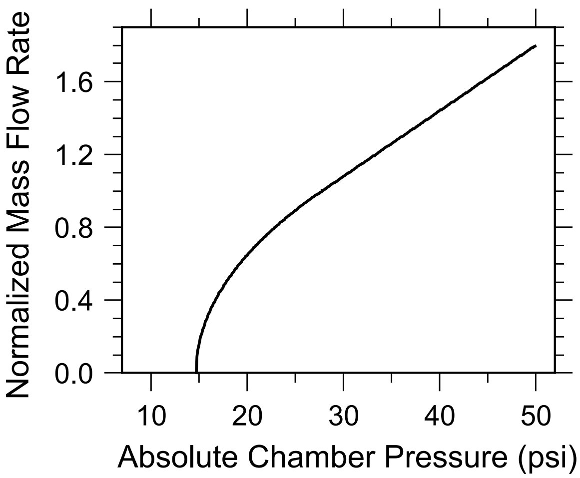

Figure 7. A graph of normalized mass flow rate as a function of absolute chamber pressure, spanning both sub- and super-sonic flow regions.

In the cases presented above, the transition from sub-sonic to supersonic flow happens at (i.e., the critical pressure is) approximately 27.8 psia. The above curves are generic; the exact shapes of the mass flow curves depend on other factors such as the composition of the gas, gas temperature, and such.

As can be seen from Figure 7, while there is a decrease in the slope of the mass flow curve with increasing pressure—until the flow reaches sonic velocity—clearly the mass flow rate continues to smoothly increase even after the flow has reached sonic velocity in the throat section. Accordingly, there is no basis for invoking a theory of choked flow as the reason for the precipitous rise in mortar pressure during the course of firing aerial shells.

The author used the following references, but a quick perusal of technical library shelves will show this list is far from exhaustive.

General References

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