Communications

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A Simplified Method for Determining the Strength of a Tube Subjected to Internal Pressure

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When a tube is subjected to an internal pressure higher than the external pressure, the material of the tube becomes stressed in several ways. For most fireworks-related devices, usually the most important of these is the hoop, or tangential, stress. This stress places the material of which the tube is constructed in tension along the circumference of the tube. The name hoop stress comes from the similarity to the tension placed on a hoop around a barrel.

In the equations below the symbols used have the following meanings:

ID = internal diameter
OD = external diameter
S = hoop stress
P = pressure
T = thickness of tube wall

In the case of a "thin walled" tube, where the ratio of the wall thickness to the internal diameter is 0.1 or less, a simple equation for hoop stress is:

$$S = \frac{ID \times P}{2 \times T}$$

However, a more general method is to use Lamé's Equation:

$$T = \frac{ID}{2}\left[\sqrt{\frac{S+P}{S-P}} - 1\right]$$

The author, who sometimes spends too much time surfing the Web, came across a paper dealing with the optimization of hydraulic cylinders to be used in high pressure research. This paper[1] presented a somewhat different form of Lamé's Equation:

$$S = \frac{P(w^2+1)}{(w^2-1)} \text{ where } w = \frac{OD}{ID}$$

which Hall credits to S. Timoshenko.[2]

In this form of the equation, it becomes obvious that it is the *ratio* of the inner and outer diameters that govern the stress at any given internal pressure and this ratio is therefore generally applicable to any internal or external diameters.

The graph in Figure 1, which indicates the hoop stress as a function of this ratio, will usually be found to be sufficiently accurate for most fireworks purposes and will help avoid tedious calculations.

Example: A mortar tube is made of a plastic having a safe stress level of 2000 psi, with an ID of 4 in. and an OD of 4.5 in. Given that the expected internal pressure generated when the mortar is fired has always been less than 200 psig, is the mortar safe?

Solution: w, the ratio of the OD to the ID is

$$4.5 \div 4.0 = 1.125$$

From the graph the stress multiplier from the graph, at a ratio of 1.125 is 4.5. Therefore, the tangential, or hoop, stress in the tube is

$$200 \text{ psi} \times 4.5 = 900 \text{ psi}$$

The mortar is probably safe to use since the allowable stress is 2.2 times the expected stress, however a thicker tube wall would yield a greater margin of safety.

References

1) H. Tracy Hall, "Hydraulic Ram Design for Modern High Pressure Devices", *The Re-*

view of Scientific Instruments, Vol. 37, No. 5, 1966, pp 568–571.

2) S. Timoshenko, *Strength of Materials*, Part II, 3rd ed., D. VanNostrand, p 208, 1956.

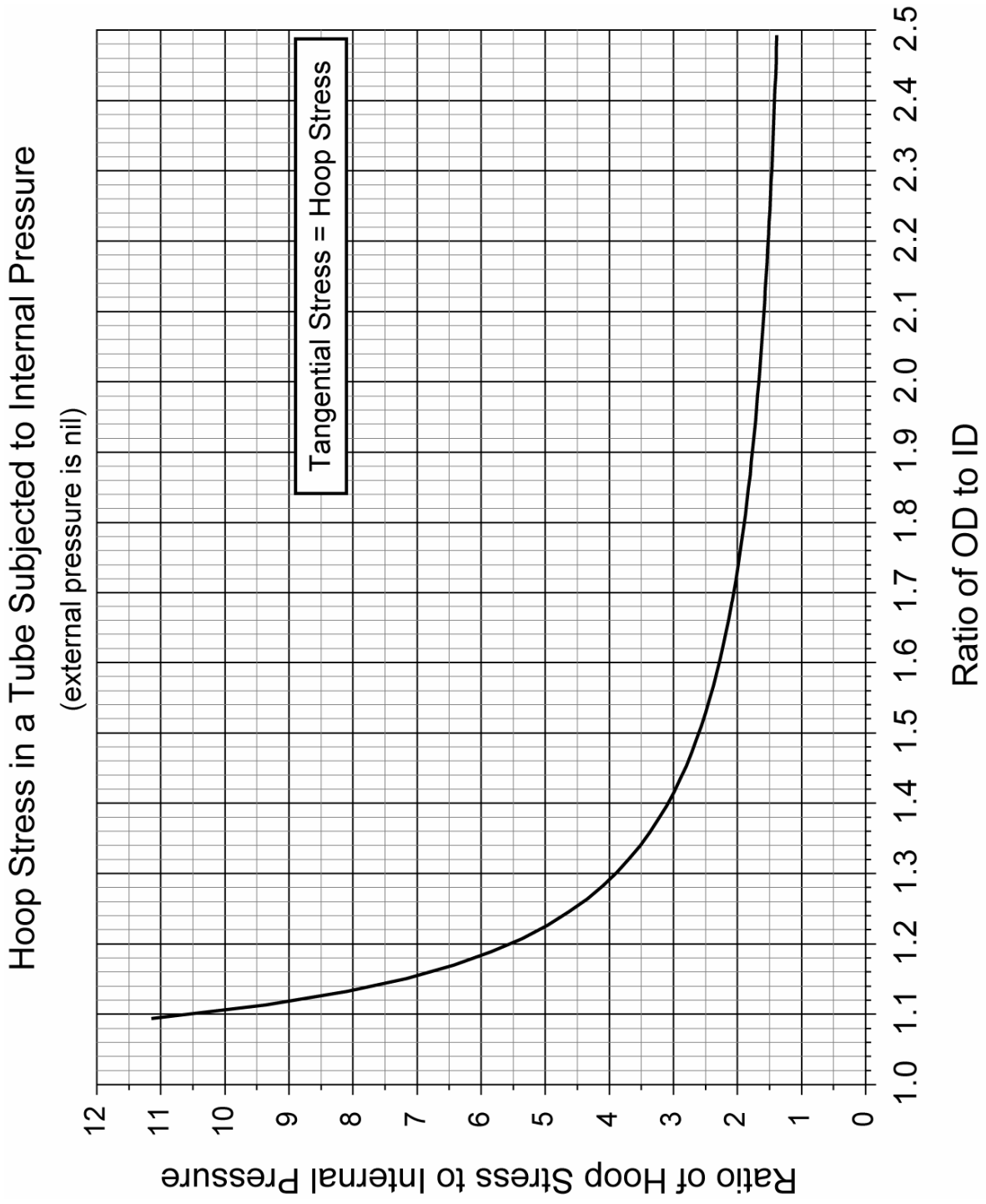

Figure 1. Graph showing hoop stress as a function of the ratio of the inner to outer diameter.