Some Properties of Explosion Generated Toroids

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ABSTRACT

The mechanism of sound production from explosion-created toroids is discussed, as well as progress in rendering them more visible. The toroids, or "smoke rings", are easily formed by exploding a small charge at the bottom of a cylindrical barrel that is open at the top. The stability, self propulsion, velocities, visibility, and sound frequencies are discussed. The dependency of these properties on the toroid production parameters is experimentally compared with theoretical predictions.

Keywords: toroid, vortex, smoke ring, explosion generated, sound emission, velocity, stability, frequency, visibility

Introduction

One of the most amazing, yet easiest to produce fireworks effects, is the "howling smoke ring". A small charge is placed at the bottom of a cylindrical barrel that is open at the top. When the charge is exploded, a smoke ring (toroid) forms at the top of the barrel and rises upward for hundreds of feet into the air at high velocity, howling like a banshee all the way up. It can last for many seconds before it vanishes. The toroid is a type of vortex in which the two ends of the vortex are joined together, forming a doughnut shaped configuration. Many of its properties are shared with other types of vortex motion in that it can contain both large amounts of stored energy and possess near stability under the proper conditions. The large energy storage has led to attempts to develop the toroid as a device to shoot down airplanes and for use as an anti-personnel weapon.[1] The most familiar type of toroid is the "smoke ring". Other vortex types that are commonly known are the tornado, the "dust devil", and airplane wake turbulence.

The production of sound by a circular toroid, or by a linear vortex, is due to the turbulence produced by the circulating flow of the gases contained in it. While this flow can be supersonic, subsonic turbulence is also quite capable of producing sound. For example, turbulence is the source of the sound produced by brass instruments. The type of horizontal vortex produced in the wake of airplanes has long been known to emit sound, at frequencies typically of 1500 Hz or lower, and this sound has been proposed as a means of detecting wake turbulence vortices at airports.[2] Toroids have been seen to form in the firing of large cannons, but the noise produced by those toroids is masked by the larger noise of the cannon. In the type of toroid produced in fireworks, a rather small explosive charge is used to produce the toroid, so that the noise it produces can easily be heard, and under the proper conditions, the toroid can easily be seen. This paper discusses the physical properties of toroids, how they can be produced with explosives, and our work on making them more visible. The mathematics concerning toroid stability and the diameter/velocity relationships are fairly involved. The article concentrates less on formal mathematics and more on the use of descriptive terms, although some mathematics is necessary to quantify toroid properties.

This paper covers several fields; pyrotechnics, acoustics, optics, and aero/hydrodynamics. Going into detailed discussions in all of those fields might overwhelm some readers and cause a loss of interest in this fascinating phenomenon. The appendix contains additional information and references in the fields of optics, aerodynamics, and hydrodynamics, for those desiring more technical information. The authors are hobbyists and possess no sophisticated instrumentation for performing measurements. Consequently, our experimental measurements were often made using techniques yielding more qualitative than precise quantitative values. The most important missing instrumentation was a

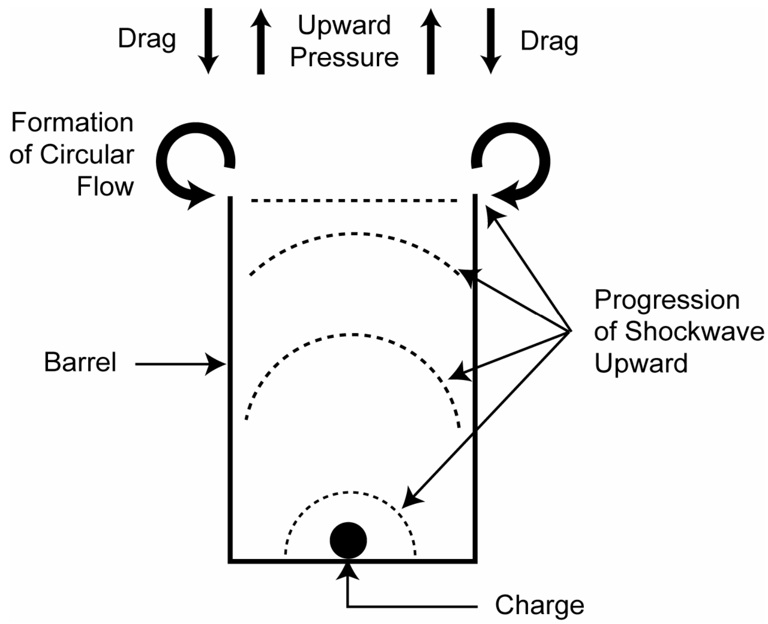


Figure 1. How the toroid is formed.

video camera, capable of directly measuring the upward velocities of the toroids and their physical dimensions versus time from formation. The experiments reported on here were performed during two time segments, separated by two years. In the first time segment, we had no video camera measurements, in the second some initial upward velocities could be measured by use of an inexpensive camcorder. Some limited correlation between those two measurements could be made. In spite of those deficiencies, we believe that our interpretations of the measurement results are reasonably consistent with the present state of understanding in the several fields.

Generation of the Toroid

If a small charge is exploded at the bottom of a cylindrical barrel that is open at the top, a shock (pressure) wave travels up the barrel as shown in Figure 1. As the shock wave travels upward, it changes from having a spherical wave front near the charge, to an almost flat wave front at the top of the barrel. When the shock wave reaches the top of the barrel, due to the high pressure inside the barrel and the low pressure outside, a circulating air current is formed at the lip of the barrel, as shown in Figure 1. This creates a toroidial smoke ring, as the effect occurs equally around the perimeter of the barrel. As the toroid rises, its shape remains relatively constant, as the centripetal forces attempting to enlarge the diameter of the circulating flow are balanced by the external aerodynamic forces acting on it.

The barrel used for most of our experiments was a "standard" American 55 gallon (200 L) steel drum. Its dimensions are approximately 23 inches (0.58 m) in diameter and 35 inches (0.89 m) in height. The explosive charge can be varied in weight and typically consists of 7 to 12 grams of a potassium perchlorate/aluminum "flash" powder, as used in fireworks salutes. The bottom of the barrel is protected by a ¼-inch (6-mm) thick steel plate.

With the dimensions of the barrel used, the ratio of height to diameter is approximately 1.5 to 1. A higher ratio would be desirable, but if the barrel diameter is kept at 23 inches (0.58 m), a longer cylinder would become difficult to handle. A diameter of much less than 23 inches would not be desirable, because the smaller diameter of the toroid produced would make it less visible. It is important to have the barrel sitting on a flat, rigid surface, or the impulse of the explosion on the bottom of the barrel could result in movement prior to the formation of the toroid at its top and interfere with the formation of the toroid.

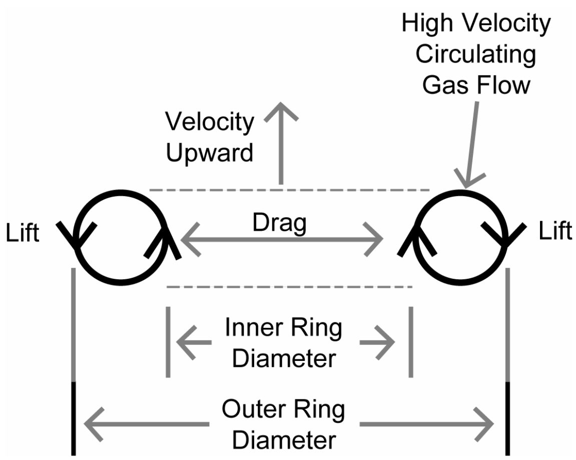


Figure 2. How the toroid propels itself upward.

Figure 2 shows the toroid rising. It propels itself upward because the downward movement of the gas at the toroid's outer diameter pushes down on the air around it, creating lift. There is a downward force produced by the upward movement at the inner diameter of the toroid, but due to the smaller circumference there, the lift contribution dominates. The sound emitted by the toroid has a principal frequency, which allows its velocity exiting the barrel to be measured using Doppler frequency shift measurements. The toroid exits the barrel at its maximum velocity, emitting a relatively low fundamental sound frequency, due to the Doppler shift. As it rises, its upward velocity continues to drop until it is practically stationary at its highest elevation, at which point it disappears. The frequency of the observed sound reaches a maximum when the smoke ring is standing still at its maximum altitude. We have measured the maximum velocity of the toroids as they exit the barrel by measuring the total Doppler frequency shift of the sound, from the frequency observed when it exits the barrel (maximum Doppler shift, minimum frequency), to the frequency emitted when it has become stationary at the top of its climb, (zero Doppler shift, maximum frequency). We assume that the intrinsic frequency of the toroid does not change during this time. The conditions necessary for this assumption to be correct are discussed in the following section.

Our Doppler measurement was done by recording the frequencies emitted and determining the frequencies exiting the barrel and at the top elevation reached, using a microphone and recorder located close to the barrel. The equation used was:

$$F_b = \frac{F_h(V_s - V_b)}{V_s} \qquad \text{Eq. 1}$$

where,

F_b = Observed frequency of toroid leaving barrel

F_h = Frequency of toroid at highest point

V_s = Velocity of sound, approx. 1100 ft/s (38 m/s)

V_b = Velocity of toroid leaving barrel

Depending on the size of the barrel and the strength of the explosive charge, velocities were measured of as low as 80 miles per hour (mph) (117 ft/s or 36 m/s), and as high as 200 mph (293 ft/s or 73 m/s). A smoke ring exiting at 200 mph is very difficult to see, as it moves so quickly that the eye has difficulty observing and tracking it, so that velocities below 100 mph (160 kph) are more desirable for fireworks purposes. The use of Doppler shift techniques to deduce the velocity of the toroid exiting the barrel was due to the simplicity of the measurement. A more desirable technique would be to measure the velocity directly, using a video camera capable of establishing a frame-by-frame time reference of the position of the toroid. We had no such equipment at the time of the Doppler shift measurements. A few video camera initial velocity measurements were made on toroids at a later date, during experiments to improve the visibility of the toroids. (See visibility of the toroids section). The flash charge used in the Doppler shift measurements was 12 grams in a two-foot (0.60 m) diameter barrel. Referring to the later video camera measurements on a reduced charge of 10 grams, also in a two foot diameter barrel, the velocity at the barrel for a 12 gram charge would be expected to be approximately 150 ft/s (102 mph or 164 kph). Our Doppler measurements for those conditions indicated an initial velocity of about 140 ft/s (43 m/s).

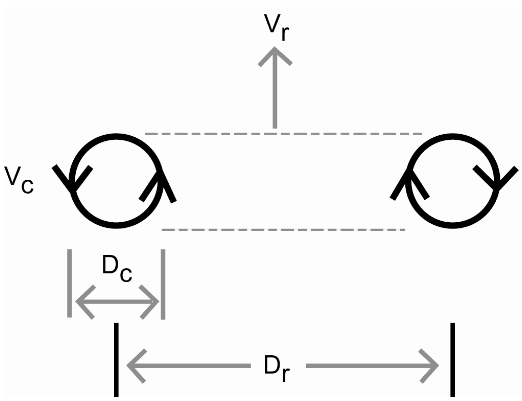
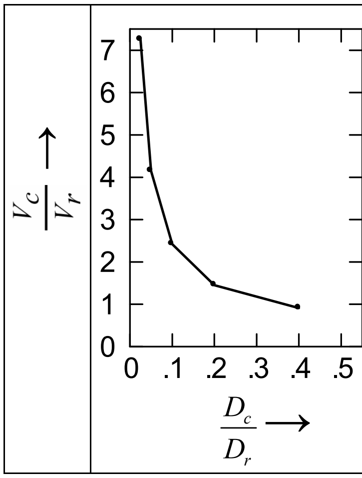


Figure 3. Drawing of toroid showing dimensions and velocities entering the stability equations.



$\frac{D_c}{D_r}$	$\frac{V_c}{V_r}$
0.025	7.25
0.05	4.15
0.1	2.42
0.2	1.45
0.4	0.91

Figure 4. A plot of the relationship between V_c/V_r and D_c/D_r

Properties of the Toroid

A. Velocity Relationships

Another drawing of the toroid is shown in Figure 3, in which the major diameter is referred to as the "ring" diameter, and the small diameter of the circulating flow is termed the "core" diameter. V_r is the upward velocity of the ring, and V_c is the circulating velocity at the perimeter of the core. According to Prandtl,[3]

$$V_r = \frac{\Gamma}{\pi D_r}\left\{\ln\frac{8D_r}{D_c} - \frac{1}{4}\right\}$$

$$\text{where } \Gamma = \oint V_c \mathrm{d}r_c \cong \pi D_c V_c \qquad \text{Eq. 2}$$

$$\text{or } \frac{V_r}{V_c} \cong \frac{D_c}{D_r}\left\{\ln\frac{8D_r}{D_c} - \frac{1}{4}\right\}$$

This equation predicts a relationship between the core and ring velocities and their respective diameters. The relationship is plotted in Figure 4. While we could not measure D_c/D_r, under our test conditions the ratio does not appear to be less than 0.1. The core velocity was therefore no more than 2.4 times the ring velocity. Since the maximum ring velocity we measured was, at most, 200 mph (320 kph), our maximum core velocity was subsonic at only 480 mph (770 kph) or less, showing that supersonic turbulence is not required to produce the sound emitted by the toroids. This relationship is plotted in Figure 4. If one assumes that both the intrinsic frequency and D_r remain constant as the toroid rises, our Doppler measurements will yield correct initial velocities. The reasonableness of this assumption will now be treated.

B. Production of Sound

Toroids having low core and ring velocities produce no audible frequencies. Those toroids move silently through the air because no turbulence is produced at the interface between the moving toroid and the still air (laminar flow). The core velocities of explosion-generated toroids are sufficiently high that a great degree of turbulence is created at the interface. If the cross-section of the core remains perfectly circular, the noise created would consist of a broad spectrum of frequencies. While the toroids we produced possessed many frequency components, one primary frequency dominated the audible emissions observed. This primary frequency would vary with experimental parameters, such as the diameter of the barrel, and the size of explosive charge used, but there was always one primary audio frequency (or narrow frequency band) generated. It is of interest to consider how this can happen.

A large number of studies have been made on audible emissions from toroids. Most of those were studies in small water filled chambers, but some also in a compressible medium such as air. We refer here only to the measurements performed in air.

Toroids have been found to exhibit instabilities under turbulent conditions, leading to a distortion in the shape of the toroid. Consider the core of a toroid in which no distortion exists (A) and a simple distortion (B). (See Figure 5.)

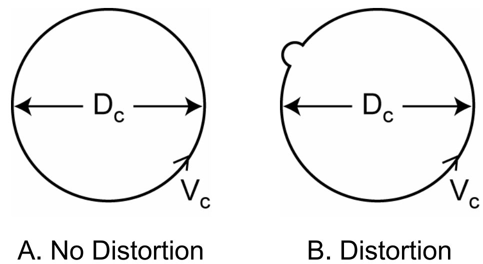


Figure 5. How sound can be generated by a distortion in the core.

A distortion, such as shown in (B), will obviously generate a primary frequency at:

$$f = \frac{V_c}{\pi D_c} \qquad \text{Eq. 3}$$

where the perimeter equals πD_c

From measurements performed in air, Zaitsev and Kop'ev[4] (also located in reference 5 on page 688) found that equation 3 is adequate to account for the dominant emitted frequency (or narrow frequency range), for a fast toroid in air.

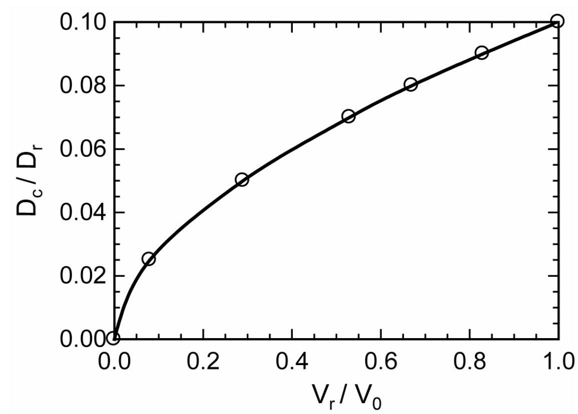
Therefore, combining equations 2 and 3,

$$f = \frac{V_r}{\pi \frac{D_c^2}{D_r}\left\{\ln\frac{8D_r}{D_c} - \frac{1}{4}\right\}} \qquad \text{Eq. 4}$$

Since we have no present capability of measuring either V_r or D_c/D_r versus time, we make the following simplifying assumptions:

1) The intrinsic frequency does not change as the toroid rises. That is the assumption made in our Doppler measurements.
2) The ring diameter remains constant with time. That appears to be true from visual observations.

If both the frequency and the ring diameter remain constant with time, then as the ring velocity decreases as the toroid rises, the ratio of the core diameter to the ring diameter must also decrease or



$\frac{D_c}{D_r}$	$\frac{V_r}{V_0}$
.1	1
.09	.83
.08	.67
.07	.53
.05	.29
.025	.08
0	0

Figure 6. Decrease in toroid core diameter with decrease in upward velocity of ring referred to initial velocity. Assumptions discussed in text.

$$\begin{aligned} V_r &= \pi f \frac{D_c^2}{D_r}\left\{\ln\frac{8D_r}{D_c} - \frac{1}{4}\right\} \\ &= \pi f D_r \left(\frac{D_c}{D_r}\right)^2 \left\{\ln\frac{8D_r}{D_c} - \frac{1}{4}\right\} \end{aligned} \qquad \text{Eq. 5}$$

This is plotted in Figure 6 for an initial ring velocity of V_0.

We now calculate the values of the parameters, using equation 5, for the following conditions:

1) The explosion of 12 grams of flash composition in a 2-foot diameter barrel.
2) The initial ratio of D_c/D_r at the barrel visually estimated to be about 0.1
3) An initial V_r at the barrel is V_0 and equals 150 ft/s (0.45 m/s) as estimated from a video camera measurement.

Table 1. Parameter Values for a Typical Rising Toroid.

$\frac{D_c}{D_r}$	D_c (ft)	$\frac{V_r}{V_0}$	$\frac{V_c}{V_0}$	V_c for V_0 = 150 ft/s (ft/s)	f for V_0 = 150 ft/s (Hz)
.1	.200	1	2.420	363.0	578
.09	.180	.83	2.178	326.7	578
.08	.160	.67	1.963	290.4	578
.07	.140	.53	1.694	254.1	578
.05	.100	.29	1.210	181.5	578
.025	.050	.08	0.605	90.7	578
0	0	0	—	0	—

Initial toroid velocity $V_r = V_0 = 150$ ft/s (45 m); $D_r = 2$ ft (0.6 m); $D_c/D_r = 0.1$; $D_c = 0.2$ ft (0.6 m).
To convert from ft/s to m/s multiply by 0.3048.

The calculated values of all parameters are listed in Table 1. The top line contains the values as the toroid leaves the barrel, while the bottom line contains the values at its highest point. The calculated frequencies in the last column are in reasonable agreement with the intrinsic measurement obtained by Doppler measurements. (550 Hz), and the initial velocity obtained by Doppler measurement (140 ft/s or 0.42 m/s).

A consistent picture emerges from Table 1 that can explain all of the phenomena observed. As the toroid rises:

1) D_c/D_r decreases, reducing lift, as the difference between the inner and outer toroid diameters becomes smaller. From Figure 3 $\left(D_r - D_c\right)$ and $\left(D_r + D_c\right)$
2) V_r decreases.
3) This process continues as the rotational energy stored in the core is drained, due to losses to turbulence, sound production, and gravitational potential energy.
4) The intrinsic frequency generated remains constant since, as the core's rotational velocity decreases, the transit time of a distortion around the perimeter of the ever-smaller core remains constant.
5) The toroid vanishes at its peak height, since the lift, D_c/D_r, and the stored energy, all go to zero.

Although this picture is logical and self-consistent, its validity should be tested by directly measuring D_c/D_r and V_r with time. That measurement is not within our capabilities at present.

Visibility of the Toroid

One problem with smoke rings is that they can be difficult to see. Not only are they rising at a great speed, but also they are normally pale white in color. They can be hard to see against the light blue color of the sky or the white color of clouds. Two approaches are useful in improving the visibility during daytime, increasing the scattering by incident sunlight and increasing the color difference between the toroid and the background by introducing color-absorbing material into the toroid.

A. Increasing the Light Scattering Power of the Toroid

The scattering power of the toroid depends on the index of refraction of the products of explosion contained within it and the size of the particles. Since we have no knowledge of the size of the particles trapped in the toroids, an exact analytical treatment of the scattering cannot be given. There are three distinct regions of light scattering, depending on the size of the particle relative to the wavelength of the light. These three regions are discussed in the appendix. In all of those cases, the effective index of refraction of the particles is important, as it distinguishes the particle from the surrounding air. For a given particulate weight trapped in the toroid however, the size is also very important in determining the total light scattering cross

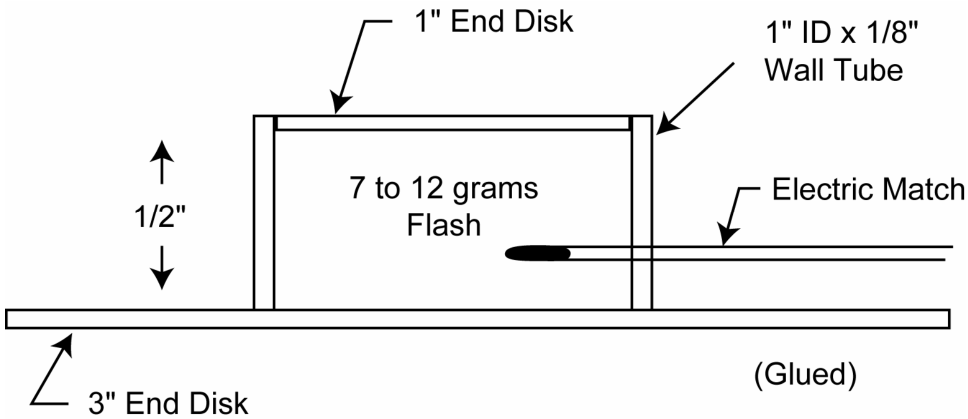


Figure 7. Assembled charge. [Note: to convert from inches to mm multiply by 25.4.]

section. Table 2 lists the indices of refraction of some typical flash powder by-products.

Table 2. The Index of Refraction of Some Flash By-Products.

Compound	Index of Refraction
KCl	1.49
Al_2O_3	1.7
MgO	1.74
TiO_2	2.5 to 2.9*

*varies with crystal structure

It is the difference between the index of refraction of the particles and the index of air (=1) that is important. From Table 2 it can be seen that the substitution of some or all of the aluminum with titanium should give the best visibility, and that was verified by our tests. Even increasing the relative amount of aluminum in the flash composition gives some improvement. Formulas that worked well are listed in Table 3.

Table 3. Flash Compositions for Increasing the Toroid Visibility.

Formula	$KClO_4$ Wt. Parts	Wt. Parts / Fuel
1	7	5 / Alcan-Toyo Al-105
2	6	3 / 'Very fine' Ti
3	50/50 mix of formulas 1 and 2	

The potassium perchlorate must be ground to an extremely fine dust for these formulas to work properly, as they are considerably off stoichiometry (the air in the barrel makes up for the missing oxygen). Alcan-Toyo Al-105 is a 6-micron, atomized aluminum; other very fine aluminum may be substituted. The "very fine" titanium was obtained from the Fire Art Corporation in Clearfield, PA. Similar particulate titanium from other sources could be substituted. The size distribution we measured for the Fire Art "very fine" titanium is listed in Table 4.

Table 4. Measured Size Distribution of the Fire Art Titanium.

Mesh Size	Pass/Stop	Wt. %	Size Micron
200	pass	100	<72
325	stop	15	>42.5
400	stop	45	>37.5
400	pass	40	<37.5

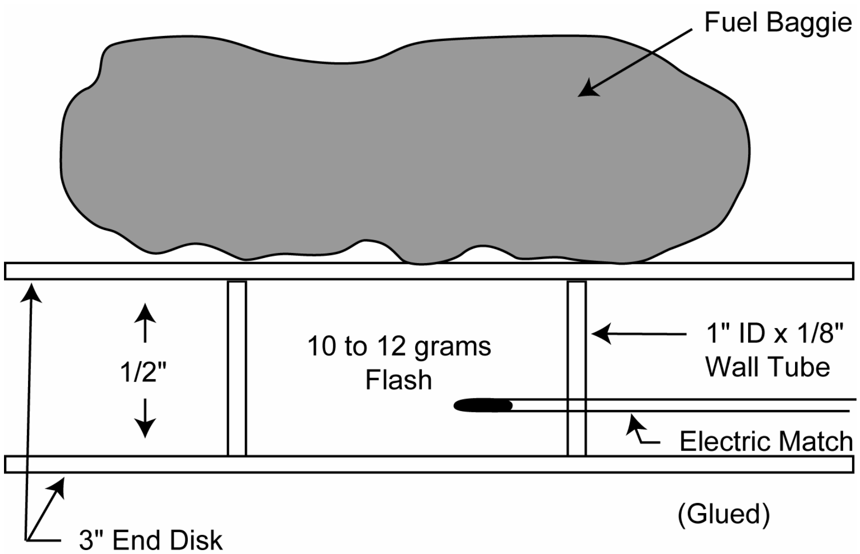


Figure 8. Plastic "baggie" full of fuel over flash configuration.
[Note: to convert from inches to mm multiply by 25.4.]

Of the three formulas given, formula 2 was the most visible, followed by formula 3, then 1. All could probably be changed to increase even further the fuel excess present, limited only by the ability of the composition to function properly. These mixtures were put into containers as shown in Figure. 7.

We also did optimization experiments in which an attempt was made to increase the visibility by incorporating material external to the charge. Two configurations were used as shown in Figures 8 and 9. In Figure 8, a plastic sandwich "baggie", which contained various fuels, was placed over the top of the charge. The hope was that some of the material in the baggie would be carried up with the shock wave, perhaps oxidizing on the way up, and become entrapped in the smoke ring to increase its visibility. The materials tried and the results are listed in Table 5.

Although some visibility improvement was seen with the "baggie" approach, we felt that much of the material was being scattered in directions that would not allow it to be trapped in the toroid, and that it was therefore being wasted. The configuration shown in Figure 9 was adopted to minimize the waste, as the material would be directed more in the direction of the toroid formation. That indeed proved to be the case, and visibility improvements equivalent to the "baggie" approach could be obtained with one third or less fuel. For example, as little as 20 grams of "very fine" titanium gave results comparable to 100 grams in the "baggie" approach. The flash compositions containing titanium worked as well as either of these approaches and used even less titanium. The configurations of Figures 8 and 9 are useful however, in evaluating the incorporation of new materials.

Table 5. Materials Used and Results of the "Baggie" Tests.

Material (100 g)	Test Results, Visual Observation
None	Standard for comparison
Lime dust	No improvement
Red Phosphorus	No improvement
#809 Dark Aluminum	No improvement
#401 Alcoa 12 micron Al.	Slight improvement in visibility
#813 Aluminum, bright flakes	Improved visibility
#105 Alcan-Toyo 6 micron Aluminum	Yet better improvement
Fire Art very fine Ti	Best improvement of all

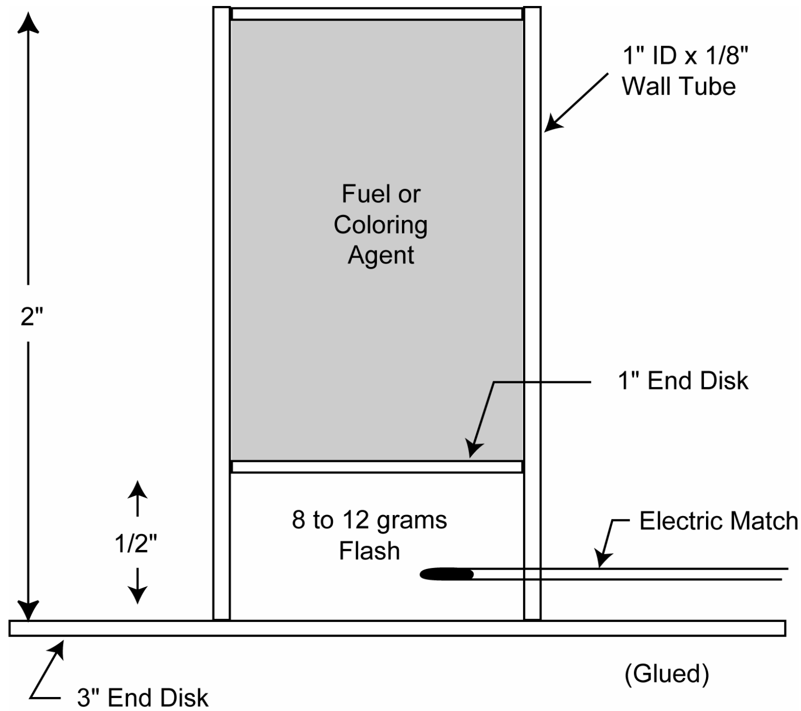


Figure 9. Coaxially stacked fuel or coloring agent over flash configuration. [Note: to convert from inches to mm multiply by 25.4.]

B. Increasing the Toroid Visibility by Introducing Light Absorbing Material into It

This method places colored absorbing material in the toroid, but it is not without problems. It not only adds coloration to the toroid but also reduces the visibility enhancement due to light scattering. Under the proper ambient conditions this trade-off is desirable. A series of experiments were run using both the "baggie" arrangement of Figure 8 and the arrangement of Figure 9. Materials used were carbon lamp black and three inorganic pigments: iron(III) oxide (red), chromium(III) (green), and an unspecified mixture of oxides of nickel, titanium, antimony (yellow). The inorganic pigments were obtained from suppliers of pigments for paints, and their particulate size distributions are unknown. The finest appeared to be the red iron(III) oxide. Observations were made both visually and recorded using a video camera. The two methods supplemented each other as some effects were most easily seen by eye and some by the camera. The camera witness also had the advantage that it could be played back, over and over, to establish the exact results. One very unexpected result was that two distinct toroids would often be formed, one traveling at high speed, and a second toroid traveling at a much slower velocity. Both toroids contained some of the color absorbing material. The results are listed in Table 6.

Only in test 7 were both "slow" and "fast" colored toroids seen. Sometimes only a "slow" colored toroid was seen, and sometimes only a "fast" colored toroid was seen. In a few tests apparently no "fast" toroid was formed, as evidenced by the lack of the sound normally produced. The "fast" colored toroids appeared to have the velocity typical of other toroids, while the "slow" toroids were moving much slower and emitted no sound. A moderate wind was blowing during the tests and the "slow" toroids would be quickly deflected away from the path of the "fast" toroid, and out of the range of the camera, so that they were best seen visually. We estimate that their velocities were about one tenth as fast as the velocities of the "fast" toroids and did not last nearly as long.

There are two plausible explanations for the formation of the "slow" colored toroids. One is that the pigments contained a large distribution

Table 6. Results of the Coloration Tests.

Test	Flash (g)	Color	(g)	Observation
1	10	Black	(20)	Fast black toroid seen with loud sound
2	10	Red	(20)	Fast faint red colored toroid with fair sound
3	10	Green	(20)	Slow green toroid, faint sound /fast toroid
4	7	Black	(20)	Fast toroid, no color, loud sound
5	7	Black	(10)	Slow black toroid, faint sound/fast toroid
6	7	Red	(10)	Slow red toroid only, no sound or fast toroid
7	7	Red	(20)	Slow red toroid, faint fast red toroid
8	7	Yellow	(10)	Fast yellow toroid only, fair sound
9	7	Green	(10)	Slow green toroid, faint sound/fast toroid
10	7	Red	(12.5)	Slow red toroid only, no fast toroid

Tests 1–9 used the arrangement of Figure 8. Test 10 used the arrangement of Figure 9.

of particle sizes and only the smaller ones could remain stably within the "fast" toroids. The larger ones would be thrown out, forming a somewhat larger, slower toroid. Another is that, since a great excess of pigment was present in the air following the explosion, with little actually becoming incorporated into the "fast" toroid, a pressure wave from the top of the barrel would be reflected to the bottom of the barrel and rise again, forming the "slow" toroid. Exactly how the "slow" toroids are formed still remains a mystery.

The coloration tests show that light-absorbing material can indeed be incorporated into the toroids. A more efficient approach to utilizing the material is needed, as very little of the material was actually incorporated. Finer pigment particle sizes would be desirable, and the explosive charges probably need to be increased in strength, as evidenced by the failure to produce "fast" toroids in some cases. More experiments should be done utilizing organic dyes, as their molecular structures are typically only about .01 microns long and would easily be stable within the toroids, assuming that they survive the combustion from the explosion.

Future Work

We intend to extend this work to utilizing organic dyes for daytime viewing. No work has yet been done on incorporating light emitting material for making the toroids visible at night. Two approaches that could have utility are the use of fluorescent organic dyes, excited by an external source of illumination, and chemiluminescent dyes. It is also possible that materials such as lampblack could leave a residual glowing effect if they are incorporated while burning. If greatly enhanced visibility of the toroids can be obtained, we may attempt to measure the toroid velocities directly and V_c/V_r as the toroids rise, using a higher quality video camera.

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References

1) D. Pasternak, "Wonder Weapons", *US News and World Report*, July 7, 1997, pp 41 and 45.
2) T. S. Perry, "Tracking Weather's Flight Path", *I. E. E. E. Spectrum*, Sept. 2000, p 42.
3) L. Prandtl, *Fundamentals of Hydro and Aeromechanics*, Dover Publications, 1957, p 212. [The equation referenced in the text appears much earlier in a translation of a paper by Helmholtz done by Sir W. Thomson that appeared in *Phil. Mag.*, Volume 33, No. 4 (1867) p 511.]
4) M. Yu Zaitsev and V. F. Kop'ev, *Akust. Zh*, Vol. 39, No. 6(1993) p 1068. [*Acoust. Phys.*, Vol. 39 (1993) p 562.]
5) V. F. Kop'ev and S. A. Chernyshev, "Vortex Ring Oscillations, the Development of Turbulence in Vortex Rings and the Generation of Sound", *Physics-Uspeckhi*, Vol. 43 No. 7, 2000, pp 663–690. English translation available from http://ufn.ioc.a.c.ru/abstracts/abst 2000 /abst.007.html
6) *American Institute of Physics Handbook*, D. E. Gray, Ed., McGraw Hill Inc., 1986, Sect. 6 p 7.
7) *Handbook of Optics*, W. G. Driscoll, Ed., McGraw Hill Inc., 1978, Sect. 14, p 11.
8) *Handbook of Laser Science and Technology*, *Vol. 4, Optical Materials*, Part 2, M. J. Weber, Ed., CRC Press, 1986, p 139.

Appendix

A. Aero- and Hydrodynamics

Kop'ev and Chernyshev[5] published a review paper on the generation of sound by toroids. Much of the experimental research reported there concerns toroids generated in an incompressible medium (water). Toroids can easily be made visible in water by launching them from a dyed region into a clear one. Studies on sound generation in a compressible medium (air) rely on sound detection more than visual detection. These studies are typically conducted in rather small measurement chambers, to facilitate measurements. By contrast, our measurements extend over hundreds of feet and involve toroids rising upwards. Since our measurement results were more difficult to obtain, we rely heavily on the "small chamber" work for analogy. The work of Zaitsev and Kop'ev[4] in an air chamber showed that the primary audio frequency range generated is due to the simple core rotational frequency of a core deformation. Much more complex core deformations have been observed in liquids, also capable of producing sound, such as those corresponding to the set of Bessel function modes (eigen values) of the toroids, due to their circular geometry. Drawings of some of these can be seen in reference 5 on pages 675 through 677. These modes involve distortions of the core such as a periodic bulging of the toroid at some sections, and a narrowing at others, or a periodic movement out of the plane of the toroid in some sections, and an opposite motion in others. The sound frequencies produced by the Bessel modes are much lower than the core rotational frequencies, but they could well have been observed by us. In every toroid we produced, a low frequency modulation of the primary frequency was present (a "wow"). Since we possessed no audio spectrum analyzer, an accurate measurement of the frequency(s) of these Bessel modes could not be measured, but they were below 10 Hz in frequency. Although the Bessel modes were observed in water chambers, analogous phenomena should appear in air also.

B. Optics.

The text treated the visibility of the toroids as a simple matter of light scattering and/or absorption, due to the index of refraction of the entrapped particles differing from the index of air. While that contains a good deal of truth, it is a simplification. The values of the index of refraction that we quoted were for well-defined, solid crystalline materials. Much of the scattering or absorbing material trapped in our toroids is anything but well defined. The products of explosion, such as TiO_2, have only microseconds to form as particles, and are certainly more amorphous than crystalline. While their effective index of refraction is certainly related to that of crystalline material, it surely differs. That is only one of many complicating issues. Another is that the index of refraction is a complex number in the case where the particle also absorbs some of the incident light.

The actual index is $N = n - ik$, where n is the index in the absence of absorption, and k is the so-called "extinction" coefficient, related to absorption, see reference 6. Three regions of scattering can be easily treated if the scattering particles are "well defined". For wavelengths much smaller than the dimensions of the particle, scattering can be treated as reflective or refractive scattering, where light is reflected or refracted from the particles. For a simple case of normal incidence on a well defined, geometrically shaped particle, the reflectance, P, is

$$P = \frac{\left(n_0 - n_1\right)^2 + k_1^2}{\left(n_0 + n_1\right)^2 + k_1^2} \qquad \text{Eq. 6}$$

For reflection from a particle of index $N = n - ik$, where in air, $n_0 = 1$. For light incident at angles off normal, the reflectance is higher, and refraction within the particle causes light to leave from its sides, reducing the transmitted intensity further. If one factors in the very irregular shapes that our trapped particles certainly possess, the calculation becomes intractable. Yet, the process is index of refraction dependent.

Light scattering from particles whose wavelength is comparable to the size of the particle is called "Mie" scattering.[7,8] Again, the Mie theory treats scattering from simple geometrical shapes, such as spheres. In spite of the complicated geometry of our trapped particles, Mie scattering still shows an increase with an increase in the index of refraction of the particles.

In the final limit, where the dimensions of the particles are much less than the wavelength of the light, the theory of Rayleigh applies,[7,8] in which the scattering power increases as

$$\frac{1}{\lambda^4}$$

where λ is the wavelength as the wavelength of the light approaches the dimensions of the particles. Here also, the scattering strength depends on the index of the particles.

The exact calculation for the scattering and absorbing power of our ill-defined particles is therefore not within reach. What remains, however, is that the index of refraction is an important parameter in all cases. Said another way, if the index of refraction of the particles was identical to that of the surrounding air, the light radiation would not know of the presence of the particles and no scattering would occur. The index values quoted in the text therefore have relative importance in the total removal of incident light, regardless of the theoretical treatment.