Peak In-Mortar Aerial Shell Accelerations

K. L. and B. J. Kosanke

PyroLabs, Inc., 1775 Blair Road,
Whitewater, CO 81527, USA

ABSTRACT

Internal mortar pressures were measured for a range of somewhat typical fireworks aerial shell firing conditions. These data were used to determine the peak shell accelerations produced during firing. Under the conditions investigated, peak aerial shell acceleration ranged from approximately 4 to 16 km/s^2 (400 to 1600 times the acceleration due to gravity) and appeared to be mostly independent of nominal shell size.

As a check on the acceleration results, the same mortar pressure data were used to calculate aerial shell muzzle velocities. These were found to be in close agreement with measured velocities.

Keywords: fireworks, aerial shell, acceleration, muzzle velocity, mortar pressure, pressure impulse

Introduction

For safety reasons, a fireworks display operator needs to know that aerial shells leave the mortar at high speed. Further, it is important to know approximately how far the aerial shells can travel. However, it is not important for the operator to know the rate of acceleration of aerial shells within mortars as they are fired. Similarly, except to know that the acceleration is great and the resulting inertial forces on the shells are large, the shell manufacturer does not need detailed knowledge of the magnitude of aerial shell acceleration. Nonetheless, it is sometimes a topic of discussion, and knowledge of these accelerations would satisfy the curiosity of a number of individuals. This short article is intended to help satisfy that curiosity.

Several years ago data was collected, albeit for another purpose, that can be used to calculate the acceleration of aerial shells while being fired from mortars. These data are internal mortar pressures as a function of time for various shell parameters (e.g., size and shape, lift type and mass, and shell mass). At the same time, the muzzle velocity of the shells was measured and can be used as a check on the calculated shell accelerations. Some examples of the basic data and the results produced are presented in this article.

Background

If the forces acting on a body are known, it is a simple matter to calculate the acceleration produced. Pressure has the units of force per area; for example, newtons per square meter (also termed pascals and abbreviated Pa). Accordingly, the force (F) acting on an aerial shell with a known cross-sectional area (A) perpendicular to the pressure gradient, when experiencing a pressure difference (P) between one side and the other, is[1]

$$F = P \cdot A \tag{1}$$

Then simply by rearranging Newton's second law of motion, and knowing the mass (m) of the aerial shell, the acceleration (a) it experiences can be calculated as

$$a = F/m \qquad \text{or by substitution} \tag{2}$$

$$a = P \cdot A/m \tag{3}$$

Figure 1 is an example of the pressure measured inside a mortar as a shell is being fired. Because the pressure is not constant during the firing, neither is the acceleration of the shell. Nonetheless, equation 3 accurately predicts the acceleration at every instant, providing the mortar pressure at the same instant is used. Thus, the shell's acceleration reaches a maximum when the mortar pressure peaks, and this peak acceleration can be calculated using equation 3.

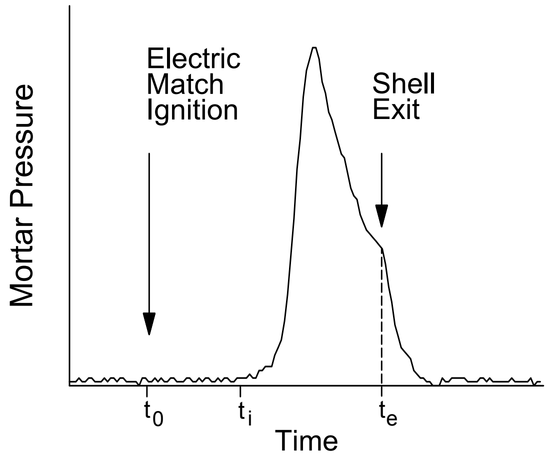

Figure 1. Typical internal mortar pressure during the firing of an aerial shell.

In the same tests where mortar pressures were measured, aerial shell muzzle velocities were also measured. This provided an opportunity to indirectly confirm the accuracy of the peak acceleration determinations by using the mortar pressure data also to predict the measured muzzle velocities.

In general, for any body, its change in velocity (v) in response to a time dependent acceleration can be represented by

$$v_f - v_i = \int_{t_i}^{t_f} a(t)\ \mathrm{dt} \qquad (4)$$

where the subscripts i and f are for initial and final values. For an aerial shell initially at rest (stationary), substituting for acceleration using equation 3, and integrating over the time of exposure to the pressure in the mortar equation 4 becomes

$$v_m = A/m \int_{t_i}^{t_e} P(t)\ \mathrm{dt} \qquad (5)$$

where v_f is now muzzle velocity (v_m) and t_f is now the time of exiting (t_e), see Figure 1.

The integral in equation 5 is usually referred to as pressure impulse (I_p). In these tests, values for the pressure impulse were determined and used to calculate the aerial shell muzzle velocities from equation 6.

$$v_m = A/m \cdot I_p \qquad (6)$$

Experimental

For uniformity, all of the test shells for this project were assembled using molded plastic shell casings. Nominal shell size ranged from 3 to 8 inches. Most shells were spherical in shape, but some 3- and 4-inch shells were cylindrical.

Table 1. General Test Shell and Mortar Information.

Nominal Shell Size (inches)	Mortar Diameter (mm)	Mortar Length (m)	Shell Shape	Shell Diameter (mm)	Shell Mass (g)
3	79	0.51	Spher.	66	135
			Cylin.	67	180
4	103	0.61	Spher.	95	350
			Cylin.	92	500
5	129	0.76	Spher.	119	620
6	154	0.76	Spher.	144	1140
8	203	0.91	Spher.	193	2700

To convert millimeters to inches, divide by 25.4.

To convert meters to inches, multiply by 39.4.

To convert grams to pounds, divide by 454.

Table 2. Test Shell Firing Results.

Nominal Shell Size (inches)	Shell Shape	Lift Mass (g)	Peak Pressure (kPa)	Pressure Impulse (kPa·s)	Measured Velocity (m/s)	Calculated Velocity (m/s)	Peak Acceleration (km/s^2)
3	Spher.	28	430	4.5	80	85	8
	Cylin.	28	500	5.0	90	95	10
4	Spher.	28	210	3.2	65	65	4
		46	660	5.9	125	125	14
	Cylin.	50	880	7.9	110	105	12
			1200	8.0	110	105	16
			970	7.9	100	105	13
			770	7.5	100	100	10
5	Spher.	50	610	5.9	100	105	11
6	Spher.	85	680	7.4	110	110	10
8	Spher.	155	830	11.2	120	125	9

To convert grams to ounces, divide by 28.3.

To convert kilopascals to pounds per square inch, divide by 6.89.

To convert meters per second to feet per second, multiply by 3.28.

In an attempt to have the spherical shells perform in a similar manner to typical oriental shells, the lift powder used was a fairly homogeneous blend of powder harvested from a collection of shells manufactured in China. The lift powder for the cylindrical shells was 2F fireworks Black Powder manufactured by Goex.[2] The air temperature at the time of firing ranged from 21 to 27 ºC (70 to 80 ºF). The tests were conducted at about 1400 m (4600 ft) above sea level, resulting in air pressure of approximately 850 mbar. Additional mortar and shell test information is provided in Table 1.

All mortars were steel with piezoelectric pressure gauges installed in the mortar plug. In this way the internal mortar pressures were measured as the shells were fired.[3] The mortars were also fitted with a series of trip wire sensors to detect the passage of the shell after exiting the mortar. Signals from the trip wires controlled a series of time counters to produce the data used to calculate velocities of the shells as they exited the mortar.[4]

The test results are reported in Table 2. In each case, the peak mortar pressure reported was the highest value from the digital pressure data. Pressure impulse is the sum of the pressure data, starting from the first sign of pressure rise (t_i) and ending at the point of shell exit (t_e) (such as identified in Figure 1). The measured velocity of the exiting shell was determined by noting the time taken for the shell to travel a known distance after exiting the mortar. The calculated shell velocity was determined by substituting the measured pressure impulse and the known cross-sectional area and mass for the aerial shell into equation 6. The peak shell acceleration was determined from equation 3, using the measured peak mortar pressure.

To be consistent with the general reliability of the data, in Table 2 peak pressures were reported to the nearest 10 kPa, pressure impulses were reported to the nearest 0.1 kPa·s, measured and calculated muzzle velocities were reported to the nearest 5 m/s, and peak accelerations were reported to the nearest 1 km/s^2.

Discussion

The aerial shells had been assembled such that their mass, the type and amount of lift powder, and the mortar specifications were fairly representative of typical aerial shells. However, caution is warranted in applying the results of these tests in situations where any of the conditions are different.

An examination of the results for the series of 4-inch cylindrical shells provides an indication of the general reliability of these data. Note that while the peak pressures (and peak accelerations) for these firings varied considerably, the pressure impulses (and thus muzzle velocities) were in relatively close agreement. The authors have seen this same type of large variability in peak mortar pressure, yet reasonably consistent overall performance, in numerous other confined-combustion measurements. The reason for this effect is not clear but is suspected to be the result of small dynamic differences in the ignition and initial flame spread within the pyrotechnic charge (an interesting subject, but beyond the scope of this article.)

There was relatively close agreement between measured and calculated shell muzzle velocities, not only for the 4-inch cylindrical shells, but all others as well. Further, the muzzle velocities were reasonably close to 100 m/s (330 ft/s), regardless of shell size. This is consistent with the results reported by Shimizu,[5] Contestabile,[6,7] and in unpublished results of the authors. Thus there is a reasonably high degree of confidence in the reported results.

The maximum shell accelerations typically ranged from 8 to 12 km/s^2 and appear to be mostly independent of nominal shell size. In part, the 4 km/s^2 value reported in Table 2 was a result of using a smaller than normal amount of shell lift powder. However, it may also be a reflection of the widely varying peak pressures thought to result from the differences in ignition and flame spread mentioned above. Similarly, the 14 and 16 km/s^2 values may again be the result of these same differences.

These peak acceleration results can be put into perspective, recalling that the acceleration due to gravity is 9.8 m/s^2. Accordingly, at their maximum acceleration, these somewhat typical aerial shells were experiencing approximately 400 to 1600 times the acceleration due to gravity. Obviously, this produces powerful forces on the contents of the shell (so-called set-back forces) and indirectly on the shell's casing as well. For example, in the relatively new "Lampare" style aerial salutes (maroons), there is generally a container of liquid fuel, combined in some fashion with a charge of flash powder. Consider a liquid fuel with a density of 0.85 g/cc that is placed in a container with a height of 150 mm (about 6 inches). If that shell is propelled such that it receives the peak acceleration seen in the tests reported above, the liquid pressure at the bottom of the container would range from 0.5 to 2.0 MPa (70 to 290 psi). Thus it clear why some fuel containers fail catastrophically during shell firing and why the fuel containers typically are strongly encased.

Acknowledgements

The authors wish to thank Scot Anderson, Wes Smith and Larry Weinman for their review of an earlier version of this article.

Notes and References

1) This is a simplification in that things such as drag forces produced by the combustion gases rushing past the exterior of the shell are not considered. However, this is not expected to introduce a significant error in the results being reported.
2) Goex, Inc., PO Box 659, Doyline, LA 71023, USA.
3) The pressure gauges were PCB Piezotronics 101A04, connected to PCB Piezotronics 480D06 amplifying power supplies. The data were recorded using Fluke Scopemeters model 95 or 97, then transferred to a computer for processing.
4) K. L. and B. J. Kosanke, "Measurement of Aerial Shell Velocity", *American Fireworks News*, No. 157, 1994. This article was also reprinted in *Selected Pyrotechnic Publications of K. L. and B. J. Kosanke, Part 3 (1993 to 1994).*
5) T. Shimizu, *Fireworks, From a Physical Standpoint*, Reprinted by Pyrotechnica Publications, 1985, p 183.
6) E. Contestabile, "A Study of the Firing Characteristics of High Altitude Display Firework Shells", CANMET, MRL 88-020 (OPJ), 1988.

7) E. Contestabile, T. R. Craig, R. A. Augsten and R. R. Vandebeek, "Evaluation of Large Diameter Fireworks Shells and Mortars", CANMET, MRL 90-41 (OP), 1990.