This article, which originally appeared in *High Power Rocketry*, May 1995, p 20, has been updated.

The Nitrous Oxide Hybrid Rocket Motor

"I might construct a rocket, in the form
Of a huge locust, driven by impulses
Of villainous saltpeter from the rear,
Upwards by leaps and bounds"

Cyrano in Cyrano de Bergerac, Act III

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Introduction

High power model rocketry has its charms, besides the sound of the outward bound. Flying a vehicle to altitude, keeping it together near Mach and finding it (after it has totally gone out of sight!)... in high power rocketry that can be quite a challenge! Recent years have seen the introduction of electronic devices both for aerobraking deployment and sequencing. We have seen high tech building materials, such as carbon composites, in high power rocketry (HPR) models. We have also seen a growth in sophistication in the standard solid rocket motors, especially with the introduction of reloads. Growth of the hobby has also drawn the attention of more regulatory agencies to solid rocket motors. Time has come for the introduction in HPR of a new propulsion technology.

Modern solid rocket motors employ propellants that insulate the walls from hot combustion products. The propellants usually are a mixture of oxidizer crystals, such as ammonium perchlorate held in a matrix of synthetic rubber (or plastic) binder along with an additive like aluminum powder. A solid motor allows for possible thrust vector control but it is difficult to throttle. Solids are relatively cheap and simple and have been very attractive for military purposes. Liquids are still the motor of choice for heavy duty orbital supply and have a long history. Though we may see liquid rocket motors move from the realm of commercial, military and amateur rocketry to HPR, it is probably a long way off. For high power model rockets there is an attractive alternative: the hybrid rocket motor.

Hybrid Rocket Motor Operation

The hybrid rocket motor usually employs propellants in two different states of physical composition. The prevalent concept is a solid fuel, such as a polymer, and a liquid or gaseous oxidizer such as oxygen (O_2), hydrogen peroxide (H_2O_2) or nitrous oxide (N_2O). Figure 1 is a schematic diagram of a hybrid motor. A pressure or pump system feeds a liquid or gaseous oxidizer into the combustion chamber, which contains the fuel as a solid component. The solid grain in Figure 1 has a single hollow circular cylinder as a flame channel called the grain port. Once ignition is initiated combustion products then converge toward the nozzle throat where they attain the speed of sound and expand in the diverging section of the nozzle reaching supersonic speeds. (One can turn the system around and have a liquid fuel and solid oxidizer, but in general this system has some disadvantages.)[1]

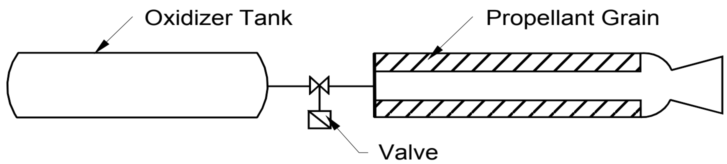

Figure 1. A schematic diagram of a hybrid motor.

A more detailed elaboration of the combustion process and flame structure is shown in Figure 2. Combustion in a hybrid rocket motor differs substantially from that in a solid or liquid fuel rocket. The oxidizer, after having been turned into a mixture of droplets and gasified liquid by the injector, streams through the combustion channel during the operation of the motor. A boundary layer is formed above the surface of the grain. This layer is fed by the oxidizer entering from the port side of the grain and by gasified fuel ablating from the grain wall.

What makes hybrid rocket motors so attractive is that they combine the advantages of both solids and liquids. There is improved safety in handling, since there is no intimate mixing of fuel and oxidizer as with solids, and the separate components can, in general, be handled with ease. Because thrust is proportional to oxidizer flow rate and internal surface area, one can consider the possibility of throttling. Another advantage is that the hybrid solid fuel component can have superior mechanical properties over the grains in a solid rocket motor.

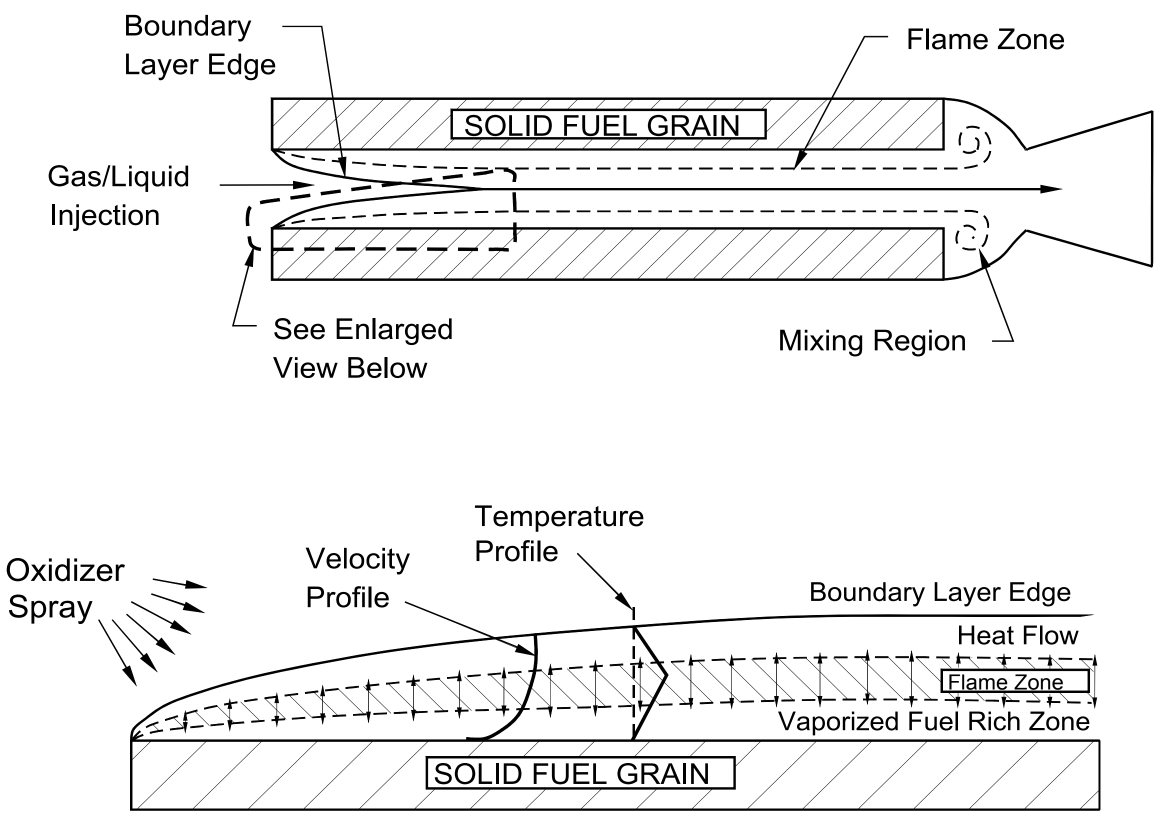

Figure 2. A more detailed diagram of the combustion process and flame structure.

To this one may add that advantages in handling and storage for hybrids bypass many of the regulatory problems at the moment with large solid motors used in high power rocketry.

Hybrids also have some disadvantages: There can be a varying specific impulse during operation and steady state combustion efficiencies that can be lower than liquids and solids. However, these are factors of more interest in very large rocket systems. The hybrid rocket motor is simpler than a liquid bipropellant rocket motor, but not as simple as a conventional solid propellant motor. Hybrids are also subject to the same combustion stability problems as solids and liquids but this probably will not affect operations as far as the high power enthusiast is concerned.

Some History of Hybrid Rocket Motors[2,3]

1929: The German film company UFA wanted a publicity stunt for their production of Fritz Lange's science fiction film *Woman In The Moon*; so they hired H. Oberth to launch a demonstration rocket. Oberth designed several rockets, but the one he finally settled on was a vehicle using a hybrid rocket motor. It was to operate on liquid oxygen as the oxidizer and carbon rods as the fuel. The combustion products were to be expelled from the top of the rocket for thrust. Some hardware was built and some tests made but the motor was never constructed or flown.

1933: The first hybrid rocket motor to actually fly was developed in the Soviet Union by S. P. Korolev and M. K. Trikhonravov as part of the GIRD program. The propellants were liquid oxygen and a colloidal suspension of benzine. A vehicle with this motor flew to 1500 meters in 1934.

1937: The German company I.G. Farben supported research by a group of their engineers, L. Andrussow, O. Lutz and W. Noeggerath who developed several hybrid rocket motors. They tested a nitrous oxide (N_2O) and coal fueled motor with a thrust of 10,000 newtons. This motor was tested with burn times of up to 120 seconds. One finds no reference to this motor ever powering a vehicle, but it is the first instance (and last! for a long time) of a nitrous oxide hybrid rocket motor.

Late 1930's to Early 1950's: The California Rocket Society and Pacific Rocket Society (PRS) built and tested a number of hybrid rocket motors. The PRS conducted a number of tests in the late 1940's using liquid oxygen as the oxidizer. Some of the fuel grains used were Douglas Fir(!), wax loaded with carbon black and synthetic rubber. A LOX-rubber [LOX = liquid oxygen] motor flew in June of 1951 reaching an altitude of 30,000 feet.

Late 1940's to present: US industrial interest in hybrids started in the late 1940's with work by engineers such as G. Moore and K. Berman at General Electric. They published an important paper about their H_2O_2 motor in the American Rocket Society's journal *Jet Propulsion* in 1956[4]. Since the mid fifties there has been a small but steady program to develop hybrids. It's not entirely clear why liquid and solid rocket motors were developed to an order of magnitude more in proportion than hybrids throughout the 1960's and 1970's.

The only hybrid rocket motor to go into production was the power plant for the Air Force target drones Sandpiper and HAST vehicles in the late 1960's. These were large throttleable hybrid rocket motors. There was also hybrid military developments in the former Soviet Union, but these are not well documented.

American Rocket Company (AMROC), founded in the mid 1980's, has now become the chief proponent and developer of hybrid rocket motors. A large 10,000 pound thrust N_2O motor was tested around 1989.[5] AMROC even demonstrated a hybrid motor that used an Italian salami as a fuel grain and liquid oxygen as the oxidizer. After the firing, the spectators ate the 'cooked fuel grain' and commented it had a delicious BBQ flavor. This, of course, conjures in the rocketeer the idea of having a launch and lunch all in one flight.

In the early eighties members of the Reaction Research Society (RRS) started to investigate the possibility of small hybrid rocket motors. In the early 1980's RRS members Bill Wood and Korey Kline speculated on the use of N_2O as an oxidizer in a small hybrid. By the early 1990's several small hybrids had been tested by the RRS, including early N_2O hybrids by Kline and an H_2O_2 hybrid built by Mark

Ventura. Independently, M. Grubelich, J. Rowlands and L. Reese[6] have also tested some small N_2O motors. Early in 1994 the company Hypertek began experiments with small hybrid motors. Later in 1994 Hypertek and AeroTech announced future availability of hybrid motors for rocketeers.

Choice of an Oxidizer

The attractiveness of hybrid motors immediately suggests itself to any high power model rocketeer who has used solid rocket motors. The fuel grain is made almost the same way except for the absence of an oxidizer. The main difference is an oxidizer that exists in a liquid or gaseous state. Since rocket motor oxidizers can have hazardous properties that range from critical injury to sudden death, one is limited when looking for a safe oxidizer for use in a hobby. Looking through a list of oxidizers, three suggest themselves: liquid or gaseous oxygen (O_2), hydrogen peroxide (H_2O_2) and nitrous oxide (N_2O). Liquid oxygen, ubiquitous in big rockets, presents cryogenic problems while gaseous oxygen has a tankage weight penalty. Hydrogen peroxide in concentrations less than 80% is a possibility, but would necessitate a separate pressurizing system and has availability problems at this concentration.

Nitrous oxide is quite interesting as an oxidizer (see Figure 3). It is in a liquid state under pressure at 70 °F. Its vapor pressure at 70 °F is about 750 pounds per square inch and has a density of approximately 47 pounds per cubic foot at this temperature and pressure. For some reasonable motor chamber pressures there is no need for a separate pressurization system because of these properties. In fact, N_2O is an example of a self-pressurizing blowdown system.

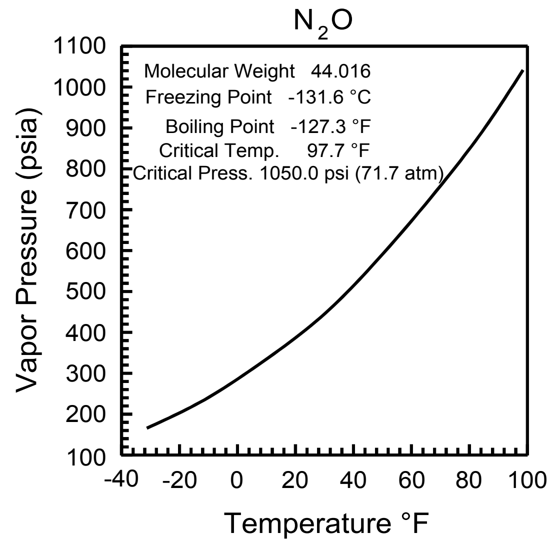

Figure 3. Nitrous oxide equation of state.

A survey of older literature turns up very little mention of N_2O as an oxidizer. The use of N_2O in a hybrid motor in Germany, as mentioned above, seems to be one of the few before some use in the Soviet Lunar program. This is because, relatively speaking, N_2O has a modest performance compared to other oxidizers. However, a graph of comparisons in Sutton[7] is quite interesting. One sees a theoretical specific impulse of over 200 for N_2O and HTPB. (See also Estey and Whittinghill.[8]) NOTE! Nitrous oxide (N_2O) is *not* the same as nitric oxide (NO_2) which is nasty stuff.

Nitrous oxide presents no significant health hazard.[9] It is nontoxic and nonirritating and has been used as an anesthetic in medicine and dentistry (it still carries the generic name of 'laughing gas'). It even forms a minor constituent of the natural atmosphere.[10] One notes, though it has been used as a mild intoxicant, it is also a simple asphyxiant. However, it is available in a 'denatured' form with approximately 200 ppm of sulfur dioxide (SO_2) added. Pure nitrous oxide is classified with a DOT label as Nonflammable gas. It will, however, support combustion. Above 572 °F, it dissociates and becomes a strong oxidizing agent. Since it is stored under pressure, one should take handling precautions as with any high pressure gas. Prudent handling of possible ignition sources in the vicinity of N_2O storage bottles is warranted.

WARNING! *Any N_2O oxidizer vessel should not be overfilled! Expansion of N_2O as a liquid may overwhelm any pressure relief system causing a system failure and an explosion.*

Denatured N_2O is available to the general public, though one should check local and state regulations. It has been a standard component of performance cars for a long time and many speed equipment hobbyists have used it for years. Your local speed shop may carry denatured N_2O. A check of local sources finds it priced in a range of $1.25 to $2 per pound.

Performance

How well does nitrous oxide perform as an oxidizer? Impulse or momentum change is more important in rating rockets motors than energy dissipated and hence a common figure of merit is the quantity specific impulse, I_{sp}, which is a function of thrust, F (in lbs) and unit weight flow rate, *dw/dt* (in lbs/s)

$$I_{sp} = F/(dw/dt) = F \bullet t/m_p g \quad \text{(in s)}$$

where t is the motor operating time, m_p is mass of propellant, and g is the acceleration of gravity. Specific impulse can be expressed in terms of thrust chamber conditions and propellant thermochemistry:

$$I_{sp} = \sqrt{\frac{2k}{k-1}\,\frac{RT_c}{g\overline{M}}\left[1-\left(\frac{p_e}{p_c}\right)^{\frac{k-1}{k}}\right]}$$

where:

- R = universal gas constant, 1544 ft-lb/mol degrees Rankine;
- g = acceleration of gravity, 32.2 ft/s^2;
- T_c = chamber temperature, degrees Rankine;
- p_e = chamber exhaust pressure, psi;
- p_c = chamber pressure, psi;
- $\overline{M}$ = mean molecular weight, lb/mol;
- k = ratio of specific heats.

Figure 4 shows the output from a computer program which computes specific impulse as a function of oxidizer to fuel ratio (o/f) for a given chamber pressure, nozzle exit pressure and thermochemical characteristics of a fuel, in this case hydroxyl/terminated polybutadiene (HTPB), and an oxidizer N_2O. One sees that the optimum oxidizer to fuel ratio is near eight and that performance is better for higher chamber pressures. For those concerned about such things let us note here the exhaust products from the theoretical calculation. One thus sees that the N_2O hybrid has a performance quite suitable for high power model rockets. (For the technically minded, a sketch of an example motor design is given in the Appendix.)

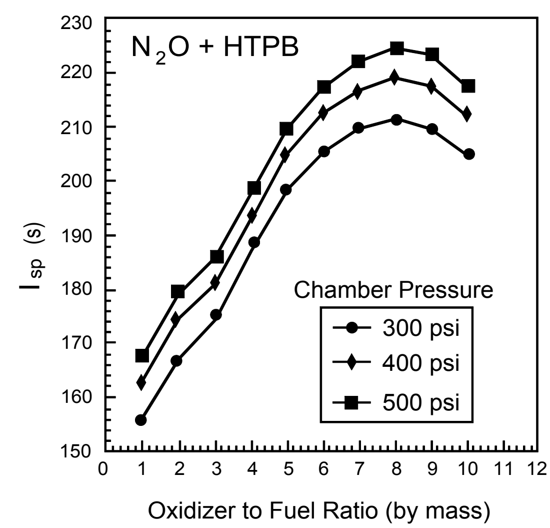

Figure 4. The output from a computer program that computes specific impulse as a function of oxidizer to fuel ratio (o/f).

The chemical exhaust products from a theoretical computation for a HTPB and N_2O combination are: 60% N_2, 19% H_2O, 19% CO_2, 1% CO and 1% other chemical exhaust products. All are normal atmospheric components. This makes hybrids even safer as emitters of polluting effluents than solid rocket motors.

Flying a Hybrid

Flying a hybrid will be almost like flying a large high power solid rocket motor. Motor prep will be similar to that of the standard reloads used now. A plastic or rubber propellant grain will be placed in the chamber and secured. There may be reusable or disposable nozzles. The whole system may use a process of loading the N_2O on the launch pad or the N_2O vessel filled separately and assembled with the motor. (We note that as with any high pressure fluid delivery system an allowance will have to be made for 'ullage', that is some volume for gas expansion is allocated. Currently, about 30% ullage is maintained by those engaged in high performance car activities.) Some provision for weighing the N_2O must also be made. Igniters may be both nonpyrotechnic and pyrotechnic, but the whole system will launch pretty much in a manner similar to that of current high power model rockets.

There are currently two manufacturers preparing to make offerings of N_2O hybrid rocket motors: Hypertek and AeroTech. We put some questions to them about their systems and here are their answers:

HPR: In what power ranges are there likely to be hybrid motors available… K and up? Smaller? Only larger?

Hypertek: Initially J to K motors, but we've designed as low as G and as high as O. It depends largely on interest expressed to us.

AeroTech: AeroTech's first RMS/Hybrid offering will be in the 54 mm H, I and J classes.

HPR: How do you mount the N_2O tank? Will most of our HPR model rockets be usable? Or more, how is the overall motor mounted?

Hypertek: The tank is mounted above the motor. The injector manifold, combustion chamber and nozzle are one-piece construction, injection molded. The motor consists, then, of two pieces; the lower (combustion chamber) portion of which is single use. Whether existing airframes can be used depends on the free space available above the motor section.

AeroTech: The RMS/Hybrid N_2O cylinder is mounted directly to the RMS casing via a special forward closure fitting. Approximately 30" of 54 mm motor mount tube will be required to accommodate the longest 54 mm casing/cylinder combination.

HPR: Are hybrid thrust profiles different from current composite motors?

Hypertek: Our motor is slightly regressive. We can vary the thrust profile to suit different missions. There will be a means of adjusting the flow rate of N_2O into the motor such that thrust can be adjusted from 35 lbs to 110 lbs on the low end, and from 60 lbs to 200 lbs on the high end.

AeroTech: AeroTech's curves will exhibit regressive time/thrust profiles.

HPR: What is the expected cost-per-flight? Give a couple of examples, for K, L, or whatever. How about the initial motor cost?

Hypertek: Expected cost per flight will be $20.

AeroTech: Expected cost per flight will be in the 1/2–1/3 range of current solid propellant RMS prices. Street price for the RMS/Hybrid "J" reload should be in the $25–30 range.

HPR: How are the tanks reloaded? Do we do that? What do hot conditions do to a N_2O tank sitting on the pad?

Hypertek: Users fill the tank immediately prior to launch. Hot weather raises the tank pressure, but not beyond design parameters. The oxidizer tank will only be filled on the pad and not transported under pressure. A pad launcher-fill system will be a part of the system. (A dump valve is part of the system to purge the tank on the pad if needed.)

AeroTech: The RMS/Hybrid N_2O cylinders can be refilled at an auto speed shop, commercial gas supplier or by the user with a 10–15 pound N_2O cylinder (also commercially available), a transfer hose and a special adaptor fitting. A gram scale will be necessary to obtain an accurate fill weight in accordance with DOT regulations. AeroTech will develop refilling procedures and instructions for use by RMS/Hybrid customers.

HPR: Given the design of the motor, do hybrid systems assume/require an independent ejection system (altimeter, timer, etc.)?

Hypertek: The recommended system is either altimeter or R/C [R/C=Radio Controlled], but other systems are being considered for release.

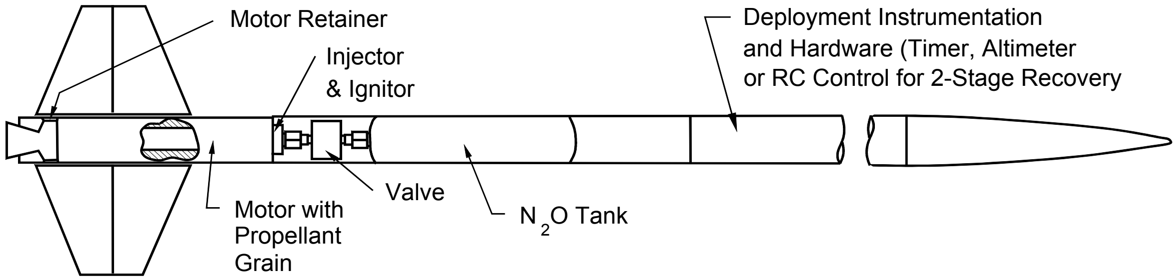

Figure 5. Sketch of a possible model hybrid motor combination.

AeroTech: The design of the RMS/Hybrid motor will require an independent recovery activation system.

HPR: What are the safety concerns of hybrid motors? How do these differ from HPR composite motors?

Hypertek: Hybrids are much safer than solids and don't share the common failure modes of solids. Our hybrid system is inert when not running.

AeroTech: The safety concerns of hybrid motors revolve primarily around the handling of compressed gases. Cylinder specifications, filling procedures, cylinder attachment, valves, contaminates and personal protective equipment are some of the issues that will need to be addressed in hybrid–oriented safety codes and motor certification standards. Of course, many of the existing safety issues regarding solid propellant HPR motors also apply, such as safe casing failure modes, repeatable performance, etc.

HPR: What is the ignition system?

Hypertek: The ignition system consists of a combination of a small electrical heating system combined with gaseous oxygen. It is a nonpyrotechnic system. A hold-down link is burned through during ignition. This integral part of the system is needed to hold the pressured tank on the pad.

AeroTech: The ignition system of the RMS/Hybrid is currently proprietary, but virtually all HPR igniters should be capable of initiating the RMS/Hybrid motor in less than one second.

Figure 5 shows a possible model-hybrid motor combination. The motor may be 'taped' in as has been done in high power rocketry, but it's recommended that a more substantial retention system be used. Remember, one will have to supply an independent deployment system as indicated.

Conclusions

It would seem exciting times are ahead for high power model rocketeers! The N_2O hybrid offers another fun aspect for flyers who like the challenge of sophisticated model rocket systems. Those flyers, interested in altitude, will have an expanded opportunity to gain those desired long-burn thrust profiles. I can even see the ambitious building on-board computers that allow for thrust programming and attainment of real trajectory optimization.

The hybrid offers a safe and reliable rocket motor for those interested in high impulse. Shipping costs of hardware and fuel grains will be small; no HAZMAT [HAZMAT=Hazardous Material] charges for inert materials! Much, much fewer regulatory problems if any at all! But better yet is the price-per-flight! J, K, L and maybe even higher total impulse motor cost per flight will be even less than what reload grains cost now! The up-front system cost will probably be in the range of the more costly reload system, but at 20 bucks and less per flight, amortization will be fast.

Acknowledgments: We wish to thank Dr. Robert Schmucker, Mark Grubelich, Hypertek and AeroTech for comments and suggestions.

Appendix A

Sketch of a Motor Design[1,2,7,8,11]

Once a propellant combination is known, and its specific impulse computed for a given chamber pressure, one may start rocket chamber layout. One is mindful of a caveat, even knowing the theoretical performance: systems efficiency and overall system interfacing will affect rocket motor efficacy. Actually motor development is more than just computing the dimensions of a motor; it takes much experimental and empirical work to make an efficient motor.

For a hybrid grain, design is of fundamental importance. In the simplest configuration, an injector sprays the oxidizer down a hollow cylinder of fuel (see Figure 2). The channel in which combustion takes place is called a 'port' just as it is in the case of a solid rocket motor. Combustion takes place in a narrow zone which is fed by gaseous decomposition of solid fuel and gases from the liquid oxidizer. The most important factor which determines fuel consumption is the velocity (*dr/dt*), which the fuel regresses in a direction at right angles to the original surface of the grain port. The laws governing the regression rate in a hybrid engine, just as in the case of a solid fuel rocket, constitute a decisive problem. The regression rate depends on various parameters, but the dominant one is the mass rate of gases (G) in the combustion channel usually defined as:

$$G = dm/dt \,/\, A_p \qquad (\text{lb/in}^2\text{-s})$$

where A_p is the port cross section area. For our purposes, the most useful functional form for dr/dt is given by the relation

$$dr/dt = a \cdot G_o^{\,n} \qquad (\text{in/s})$$

where G_o is the oxidizer mass rate, and a and n are constants. Small hybrid motors have regression rates that range from 0.1 to 0.01 in/s. From reference 11 we will take the average value of a = 0.1 and n = 0.8 for calculation purposes.

Let us look at an example motor design. Suppose you want to make a 50-lb thrust motor for a time duration of five seconds with a motor efficiency (e_f) of 90%. This makes an approximate 1100 newton second motor, or an approximate J220. Take the chamber pressure as 500 lbs/in^2 and the fuel and oxidizer to be HTPB and N_2O at a mixture ratio of o/f = 8. A theoretical thermochemical calculation shows an I_{sp} of 224.3 s, thrust coefficient (C_f) of 1.50, expansion ratio e of 5.2, and a chamber temperature of 5010 °F. The dimensions of the nozzle, injector and fuel grain are then computed from these specifications.

The nozzle configuration follows from thermodynamic theory. Using the thrust, thrust coefficient, the efficiency and chamber pressure p_c one has for the throat area

$$A_t = F \,/\, (e_f\, C_f\, p_c) \qquad (\text{in}^2)$$

and exit area is

$$A_e = e \bullet A_t \qquad (\text{in}^2)$$

From these, the throat and exit diameters can be calculated. The total weight flow rate *dw/dt* can be written as

$$dw/dt = p_c\, A_t\, C_f/(e_f\, I_{sp}) \qquad (\text{lb/s})$$

which splits into the oxidizer dw_o/dt and fuel flow dw_f/dt rates

$$dw_o/dt = (o/f)(dw/dt)/[(o/f)+1] \qquad (\text{lb/s})$$

$$dw_f/dt = (dw/dt)/[(o/f)+1] \qquad (\text{lb/s})$$

From the given operating time, the initial oxidizer and fuel weights can be calculated.

The injector is taken as a single orifice with a discharge coefficient of C_d, then if the oxidizer density is given by ρ, with the pressure drop taken as Δp (psi), then the injection velocity is given by

$$v = C_d \sqrt{(2g\Delta p/\rho)} \qquad (\text{ft/s})$$

and the area of the injector hole (or holes) is given by

$$A_o = \rho\, (dw_o/dt)/v \qquad (\text{in}^2)$$

The grain design can be computed from the following. The motor chamber diameter is taken as d_{cm} and the grain is taken with a single cylindrical port. Let the final radius of the grain port be r_f. This should be taken so a margin of protection for the outer chamber walls is given. For a given time (t) and radius of the final port, the initial port (r_p) is given by

$$r_p^{2n+1} = r_f^{2n+1} - a(2n+1)\, t(dw_o/dt/\pi)^n \qquad \text{(in)}$$

and the grain length is computed from

$$l_g = (dw_f/dt)/(2\pi r_p \rho r_r) \qquad \text{(in)}$$

where ρ is the fuel density and r_r = regression rate = dr/dt.

For the given motor parameters, the results of a calculation are shown in Table 1. Several things are of note. Because the oxidizer to fuel ratio is quite high, only a small amount of fuel is needed. Thus, one can probably make the fuel grains with quite a margin for wall protection. One can envisage that the grains would be like the standard reload grains for the solid rocket motors used now. The total oxidizer rate of nearly 0.3 pounds per second may be quite high for small solenoid valves available from, say, speed shops. In the design of a rocket motor, valving is a crucial matter to keep in mind. The valve and plumbing from the N_2O tank should be designed so that the flow is smooth and as laminar as possible. An undersized line or a valve with mismatched flow coefficient can lead to turbulence.

That constitutes an outline of motor design. Making a flight-weight hybrid motor is yet another challenge. Valves must be selected or designed with care and not weigh too much. The pressure vessels envisioned right now mostly seem to be of aluminum which can be heavy. It may be possible to use filament wound casings of composite materials. One must be mindful of system mass ratios for the kinds of motors; so it will be interesting to see if someone can build a hybrid in the E, F and G range.

Table 1. Example Motor Design.

Thrust Coefficient	1.50
Motor Efficiency	0.90
Thrust	50.00 lb
Duration	5.00 s
Chamber Pressure	500.00 psia
Chamber Temperature	5010 °F
Specific Impulse	224.3 s
Exhaust Velocity	4811 feet/s
Area Ratio	5.22
Chamber Diameter	2.50 in.
Throat Diameter	0.31 in.
Exit Diameter	0.70 in.
Nozzle Length	2.64 in.
Oxidizer Flow Rate	0.22 lb/s
Fuel Flow Rate	0.03 lb/s
Total Flow Rate	0.25 lb/s
Total Flow	1.24 lb
Total Oxidizer	1.10 lb
Total Fuel	0.14 lb
Pressure Drop Across Injector	100.00 lb/in.^{2}
Diameter of Injector	0.093 in.
Injector Speed	112.93 ft/s
Grain Length	10.71 in.
Regression Rate Exponent	0.80
Regression Rate Coefficient	0.10
Initial Port Radius	0.94 in.
Regression Rate	0.01 in./s

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7) Sutton, *op cit,* pp 502–522.

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9) Compressed Gas Association, *Handbook of Compressed Gases,* 3rd ed., Chapman and Hall, New York, 1990 pp 519–525.

10) *Ibid.*

11) P. Estey, D. Altman, and J. McFarlane, *An Evaluation of Scaling Effects for Hybrid Rocket Motors,* AIAA paper AIAA–91–2517, 1991.

Update: Both Aerotech and Hypertek introduced their hybrid motors for high power model rocketry in the spring of 1995, and both have enjoyed wide use by hobbyists for the last year and a half.

Appendix B

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HYBRID-3.BAS [listed below] is a BASIC adaptation of Al Jackson's FORTRAN program used in developing numbers for the design example listed in the appendix of his article, "The Nitrous Oxide Hybrid Rocket Motor". It can be run using GWBASIC (available on PC systems using MS-DOS through version 5) or QBASIC (on PC systems using MS-DOS version 6) or it can be run with or compiled and run with QUICKBASIC. It requires the data file MOTOR.DAT to produce output. If one wishes to run other examples, the data file or program could be easily modified to accept keyboard input of data. If one does not have access to a computer or any of the versions of BASIC mentioned, the listing can be used as a guide for manual computations. As is usually the case, it is very important to use consistent units when doing calculations. The program can also be used as a skeleton to build a much more elaborate design program. In most cases, nonprogrammers can read BASIC programs and follow the calculations done in them.

```
10 '  Al Jackson;s Fortran Program "Hybrid" converted to Basic
20 '  APRIL 9, 1997
30 '  Some changes made from original program
40 '  e.g., Conversion to Fahrenheit corrections etc.
50 '  Uses input file "motor.dat"
60    CLEAR : CLS : OPTION BASE 1: GRAVITATIONAL.CONSTANT = 32.174: PI = 3.14159
70 '         INITIALIZE INPUT VARIABLES
80   THRUST.COEFFICIENT = 0
90   SPECIFIC.IMPULSE = 0
100 THRUST = 0
110 EXPANSION.RATIO = 0
120 CHAMBER.TEMPERATURE = 0
130 CHAMBER.PRESSURE = 0
140 BURN.TIME = 0
150 MOTOR.EFFICIENCY = 0
160 OXYGEN.TO.FUEL.RATIO = 0
170 CHAMBER.DIAMETER = 0
180 INJECTOR.DISCHARGE.COEFFICIENT = 0
190 INJECTOR.DELTA.PRESSURE = 0
200 OXIDIZER.DENSITY = 0
210 FUEL.REGRESSION.COEFFICIENT = 0
220 FUEL.REGRESSION.EXPONENT = 0
230 FUEL.DENSITY = 0
240 FINAL.GRAIN.RADIUS = 0
```

```
250 '        INPUT SUBROUTINE
260 OPEN "MOTOR.DAT" FOR INPUT AS #1
270   LPRINT : LPRINT "INPUT DATA:"
280   INPUT #1, THRUST.COEFFICIENT: LPRINT "THRUST COEFFICIENT =";
      THRUST.COEFFICIENT
290   INPUT #1, SPECIFIC.IMPULSE: LPRINT "SPECIFIC IMPULSE ="; SPECIFIC.IMPULSE;
      "SECONDS"
300   INPUT #1, THRUST: LPRINT "THRUST ="; THRUST; "LBS"
310   INPUT #1, EXPANSION.RATIO: LPRINT "EXPANSION RATIO ="; EXPANSION.RATIO
320   INPUT #1, CHAMBER.TEMPERATURE: LPRINT "CHAMBER TEMPERATURE ="; CHAM-
      BER.TEMPERATURE; "DEGREES KELVIN"
330   INPUT #1, CHAMBER.PRESSURE: LPRINT "CHAMBER PRESSURE ="; CHAMBER.PRESSURE;
      "PSIA"
340   INPUT #1, BURN.TIME: LPRINT "BURN TIME ="; BURN.TIME; "SECONDS"
350   INPUT #1, MOTOR.EFFICIENCY: LPRINT "MOTOR EFFICIENCY ="; MOTOR.EFFICIENCY
360   INPUT #1, OXYGEN.TO.FUEL.RATIO: LPRINT "O/F RATIO ="; OXYGEN.TO.FUEL.RATIO
370   INPUT #1, CHAMBER.DIAMETER: LPRINT "MOTOR DIAMETER ="; CHAMBER.DIAMETER
380   INPUT #1, INJECTOR.DISCHARGE.COEFFICIENT: LPRINT "INJECTOR DISCHARGE COEFFI-
      CIENT ="; INJECTOR.DISCHARGE.COEFFICIENT
390   INPUT #1, INJECTOR.DELTA.PRESSURE: LPRINT "INJECTOR PRESSURE DROP ="; INJEC-
      TOR.DELTA.PRESSURE; "PSI"
400   INPUT #1, OXIDIZER.DENSITY: LPRINT "OXIDIZER DENSITY ="; OXIDIZER.DENSITY;
      "LB/CU FT"
410   INPUT #1, FUEL.REGRESSION.COEFFICIENT: LPRINT "REGRESSION RATE COEFFICIENT =";
      FUEL.REGRESSION.COEFFICIENT
420   INPUT #1, FUEL.REGRESSION.EXPONENT: LPRINT "REGRESSION RATE EXPONENT =";
      FUEL.REGRESSION.EXPONENT
430   INPUT #1, FUEL.DENSITY: LPRINT "FUEL DENSITY ="; FUEL.DENSITY; "LBS/CU FT"
440   INPUT #1, FINAL.GRAIN.RADIUS: LPRINT "FINAL CORE RADIUS ="; FI-
      NAL.GRAIN.RADIUS; "INCHES"
450   LPRINT : LPRINT : LPRINT "OUTPUT RESULTS:"
460 CLOSE #1
470 '        NOZZLE SUBROUTINE
480 NOZZLE.THROAT.AREA = THRUST / (CHAMBER.PRESSURE * MOTOR.EFFICIENCY *
      THRUST.COEFFICIENT)
490 NOZZLE.EXIT.AREA = NOZZLE.THROAT.AREA * EXPANSION.RATIO
500 NOZZLE.THROAT.DIAMETER = SQR(4 * NOZZLE.THROAT.AREA / PI)
510 NOZZLE.EXIT.DIAMETER = SQR(4 * NOZZLE.EXIT.AREA / PI)
520 NOZZLE.THROAT.RADIUS = NOZZLE.THROAT.DIAMETER / 2
530 NOZZLE.EXIT.RADIUS = NOZZLE.EXIT.DIAMETER / 2
540   NOZZLE.DIVERGENT.LENGTH = (NOZZLE.EXIT.RADIUS - NOZZLE.THROAT.RADIUS) /
      .267949   ' DIVIDED BY TANGENT 15 DEGREES
550   NOZZLE.CONVERGENT.LENGTH = ((CHAMBER.DIAMETER / 2) - NOZZLE.THROAT.RADIUS) /
      .57735       ' DIVIDED BY TANGENT 30 DEGREES
560   NOZZLE.THROAT.LENGTH = 0
570 NOZZLE.LENGTH = NOZZLE.CONVERGENT.LENGTH + NOZZLE.DIVERGENT.LENGTH + NOZ-
      ZLE.THROAT.LENGTH
580 '        PERFORMANCE SUBROUTINE
590 CHAR.EXHAUST.VELOCITY = SPECIFIC.IMPULSE * GRAVITATIONAL.CONSTANT /
      THRUST.COEFFICIENT
600   CSTAR = CHAR.EXHAUST.VELOCITY
610 TOTAL.PROPELLANT.FLOW.RATE = GRAVITATIONAL.CONSTANT * CHAMBER.PRESSURE * NOZ-
      ZLE.THROAT.AREA / CSTAR
620   OXIDIZER.FLOW.RATE = TOTAL.PROPELLANT.FLOW.RATE * OXYGEN.TO.FUEL.RATIO / (OXY-
      GEN.TO.FUEL.RATIO + 1)
630   FUEL.FLOW.RATE = TOTAL.PROPELLANT.FLOW.RATE / (OXYGEN.TO.FUEL.RATIO + 1)
640 '        PORT SUBROUTINE
650 SUB.ONE = FUEL.REGRESSION.COEFFICIENT * (2 * FUEL.REGRESSION.EXPONENT + 1)
660 SUB.TWO = (OXIDIZER.FLOW.RATE / PI) ^ FUEL.REGRESSION.EXPONENT
670 SUB.THREE = FINAL.GRAIN.RADIUS ^ (2 * FUEL.REGRESSION.EXPONENT + 1)
680 SUB.R = SUB.THREE - SUB.ONE * SUB.TWO * BURN.TIME
690 INITIAL.GRAIN.RADIUS = SUB.R ^ (1 / (2 * FUEL.REGRESSION.EXPONENT + 1))
700 '        GRAIN SUBROUTINE
710 PORT.CROSS.SECTIONAL.AREA = PI * (INITIAL.GRAIN.RADIUS) ^ 2
720 OXIDIZER.MASS.RATE = OXIDIZER.FLOW.RATE / PORT.CROSS.SECTIONAL.AREA
730 REGRESSION.RATE = FUEL.REGRESSION.COEFFICIENT * OXIDIZER.MASS.RATE ^
      FUEL.REGRESSION.EXPONENT
740 GRAIN.LENGTH = FUEL.FLOW.RATE / (2 * PI * INITIAL.GRAIN.RADIUS * FUEL.DENSITY *
      REGRESSION.RATE)
```

```
750 '         INJECTOR SUBROUTINE
760 GRAVITATIONAL.CONSTANT = GRAVITATIONAL.CONSTANT * 12        ' CONVERT TO
      INCHES/SECOND
770 OXIDIZER.DENSITY = OXIDIZER.DENSITY / 1728                  ' CONVERT TO LBS/CU
      IN
780 TEMP.ONE = TOTAL.PROPELLANT.FLOW.RATE
790 TEMP.TWO = INJECTOR.DISCHARGE.COEFFICIENT * SQR(2 * GRAVITATIONAL.CONSTANT *
      OXIDIZER.DENSITY * INJECTOR.DELTA.PRESSURE)
800 INJECTOR.AREA = TEMP.ONE / TEMP.TWO
810 INJECTOR.DIAMETER = SQR(4 * INJECTOR.AREA / PI)
820 INJECTOR.VELOCITY = INJECTOR.DISCHARGE.COEFFICIENT * ((2 * GRAVITA-
      TIONAL.CONSTANT * INJECTOR.DELTA.PRESSURE) / OXIDIZER.DENSITY) ^ .5
830 INJECTOR.VELOCITY = INJECTOR.VELOCITY / 12
840 '         OUTPUT RESULTS
850 LPRINT "THRUST COEFFICIENT = "; THRUST.COEFFICIENT
860 LPRINT "MOTOR EFFICIENCY ="; MOTOR.EFFICIENCY
870 LPRINT "THRUST ="; THRUST; "LBS"
880 LPRINT "BURN TIME ="; BURN.TIME; "SECONDS"
890 LPRINT "CHAMBER PRESSURE ="; CHAMBER.PRESSURE; "PSIA"
900 LPRINT "CHAMBER TEMPERATURE ="; 1.8 * (CHAMBER.TEMPERATURE - 273) + 32; "DEGREES
      F"
910 LPRINT "SPECIFIC IMPULSE ="; SPECIFIC.IMPULSE; "SECONDS"
920 LPRINT "CHARACTERISTIC EXHAUST VELOCITY ="; CSTAR; "FT/SECOND"
930 LPRINT "NOZZLE EXPANSION RATIO ="; EXPANSION.RATIO
940 LPRINT "MOTOR DIAMETER ="; CHAMBER.DIAMETER; "INCHES"
950 LPRINT "NOZZLE THROAT DIAMETER ="; NOZZLE.THROAT.DIAMETER; "INCHES"
960 LPRINT "NOZZLE EXIT DIAMETER ="; NOZZLE.EXIT.DIAMETER; "INCHES"
970 LPRINT "NOZZLE LENGTH ="; NOZZLE.LENGTH; "INCHES."
980 LPRINT "OXIDIZER FLOW RATE ="; OXIDIZER.FLOW.RATE; "LBS/SECOND"
990 LPRINT "FUEL FLOW RATE ="; FUEL.FLOW.RATE; "LBS/SECOND"
1000 LPRINT "TOTAL PROPELLANT FLOW RATE ="; TOTAL.PROPELLANT.FLOW.RATE; "LBS/SECOND"
1010 LPRINT "TOTAL OXIDIZER USED ="; OXIDIZER.FLOW.RATE * BURN.TIME; "POUNDS"
1020 LPRINT "TOTAL FUEL USED ="; FUEL.FLOW.RATE * BURN.TIME; "POUNDS"
1030 LPRINT "TOTAL PROPELLANTS USED ="; TOTAL.PROPELLANT.FLOW.RATE * BURN.TIME;
      "POUNDS"
1040 LPRINT "INJECTOR PRESSURE DROP ="; INJECTOR.DELTA.PRESSURE; "PSI"
1050 LPRINT "INJECTOR DIAMETER ="; INJECTOR.DIAMETER; "INCHES"
1060 LPRINT "INJECTOR VELOCITY ="; INJECTOR.VELOCITY; "FT/SECOND"
1070 LPRINT "FUEL GRAIN LENGTH ="; GRAIN.LENGTH; "INCHES"
1080 LPRINT "REGRESSION RATE COEFFICIENT ="; FUEL.REGRESSION.COEFFICIENT
1090 LPRINT "REGRESSION RATE EXPONENT ="; FUEL.REGRESSION.EXPONENT
1100 LPRINT "INITIAL PORT RADIUS ="; INITIAL.GRAIN.RADIUS; "INCHES"
1110 LPRINT "REGRESSION RATE ="; REGRESSION.RATE; "IN/SECOND"
1120 END
```

Sample Input file “MOTOR.DAT”:

```
1.5
224.3
50
5.22
3038.3
500
5
.9
8
2.5
.8
100
46.5
.1
.8
.033
1
```

Sample output from test data using "MOTOR.DAT" as the input file:

```
INPUT DATA:
THRUST COEFFICIENT = 1.5
SPECIFIC IMPULSE = 224.3 SECONDS
THRUST = 50 LBS
EXPANSION RATIO = 5.22
CHAMBER TEMPERATURE = 3038.3 DEGREES KELVIN
CHAMBER PRESSURE = 500 PSIA
BURN TIME = 5 SECONDS
MOTOR EFFICIENCY = .9
0/F RATIO = 8
MOTOR DIAMETER = 2.5
INJECTOR DISCHARGE COEFFICIENT = .8
INJECTOR PRESSURE DROP = 100 PSI
OXIDIZER DENSITY = 46.5 LB/CU FT
REGRESSION RATE COEFFICIENT = .1
REGRESSION RATE EXPONENT = .8
FUEL DENSITY = .033 LBS/CU FT
FINAL CORE RADIUS = 1 INCHES


OUTPUT RESULTS:
THRUST COEFFICIENT =  1.5
MOTOR EFFICIENCY = .9
THRUST = 50 LBS
BURN TIME = 5 SECONDS
CHAMBER PRESSURE = 500 PSIA
CHAMBER TEMPERATURE = 5009.54 DEGREES F
SPECIFIC IMPULSE = 224.3 SECONDS
CHARACTERISTIC EXHAUST VELOCITY = 4811.085 FT/SECOND
NOZZLE EXPANSION RATIO = 5.22
MOTOR DIAMETER = 2.5 INCHES
NOZZLE THROAT DIAMETER = .307106 INCHES
NOZZLE EXIT DIAMETER = .701655 INCHES
NOZZLE LENGTH = 2.635342 INCHES.
OXIDIZER FLOW RATE = .2201637 LBS/SECOND
FUEL FLOW RATE = 2.752046E-02 LBS/SECOND
TOTAL PROPELLANT FLOW RATE = .2476842 LBS/SECOND
TOTAL OXIDIZER USED = 1.100818 POUNDS
TOTAL FUEL USED = .1376023 POUNDS
TOTAL PROPELLANTS USED = 1.238421 POUNDS
INJECTOR PRESSURE DROP = 100 PSI
INJECTOR DIAMETER = 9.299358E-02 INCHES
INJECTOR VELOCITY = 112.9308 FT/SECOND
FUEL GRAIN LENGTH = 10.70539 INCHES
REGRESSION RATE COEFFICIENT = .1
REGRESSION RATE EXPONENT = .8
INITIAL PORT RADIUS = .9372635 INCHES
REGRESSION RATE = 1.322812E-02 IN/SECOND
```