# Observations on the Perceived Burst Size of Spherical Aerial Shells

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*ABSTRACT*

*This paper examines the effects of visual perspective on the observed size of shell bursts. The National Fire Protection Association (NFPA) codes specify minimum distances from the shell firing positions to the spectator boundaries in the United States. This paper addresses observed burst size and elevation as a function of mortar placement and spectator viewing positions, using shell burst radius and height information gathered by other researchers.*

**Keywords:** aerial shell, burst size, display, perception, mortar placement, NFPA, spectators

## Introduction

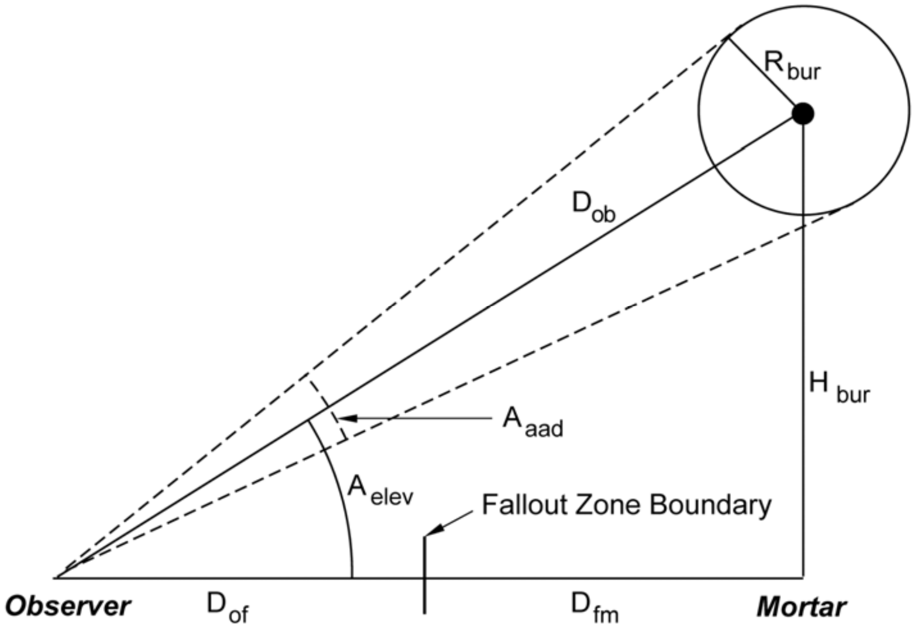
This paper grew from a debate the authors were having concerning the merits of large caliber shells in fireworks displays. It seemed obvious to us that there were distinct qualitative advantages with larger shells, such as effect duration, effect complexity, and break symmetry. It was not obvious, however, that there existed much quantitative difference in observed size for spectators (or sponsors) gathered near the fallout zone boundary, particularly when mortar placement follows the NFPA 1123 guidelines[1].

The human eye is capable of seeing light and movement nearly 90 degrees in all directions from the center of its gaze. The greatest detail, color perception, and conscious attention are generally associated with the central 25–40 degrees (diameter) of vision called the central visual field.[2,3] Bursts nearing or exceeding these angles would require conscious effort to take in the whole burst, and would contribute to the perception of a burst "filling the sky." Otherwise, two shells having the same burst angle, regardless of the actual size of the burst, would appear to have the same size. This is particularly true if smoke is present, and the observer cannot use stereo visual cues to correct for size and distance.

For example, which break would "appear larger" (assuming identical effects and manufacturers); a 10-inch shell fired 700 feet from the spectators and bursting at 1070 feet, or a 5-inch shell fired 350 feet from the spectators bursting at 700 feet? In this case it happens that the 5-inch shell actually bursts approximately 780 feet from the eye of the closest observers, and the 10-inch shell bursts approximately 1280 feet away, 500 feet further. To an observer near the fallout zone boundary, the additional distance results in the two bursts looking nearly identical in size.

In this paper, typical burst sizes of several shells launched from various NFPA–compliant locations have been translated into their apparent sizes at the observer. The term Apparent Angular Diameter (AAD) is used to denote the observed size of a shell burst. It is defined to be the angle subtended by the edge of the (spherical) shell burst, at the observer. Comparisons of the calculated AAD values for several bursts will be used to analyze observed burst sizes for several mortar placements and observer locations.

For all calculations and tables, the conversion factors used were: 1 inch = 25.4 mm; and 1 m = 3.28 feet.




$H_{bur}$ = Burst height
$R_{bur}$ = Burst radius
$D_{of}$ = Distance from observer to fallout zone boundary
$D_{fm}$ = Distance from fallout zone boundary to mortar
$D_{ob}$ = Distance from observer to center of burst
$A_{aad}$ = Apparent Angular Diameter (AAD) of burst
$A_{elev}$ = Vertical elevation angle from observer to center of burst

*Figure 1. Model used to determine AAD for various shell sizes, mortar placements, and observer locations.*

## Data Model

A simple trigonometric model was used to perform the AAD calculations. The parameters driving the model are the measured burst height ($H_{bur}$) and radius ($R_{bur}$) derived from previously published experimental results, the distance from the fallout zone to the mortar ($D_{fm}$) (which is constrained to at least the minimum NFPA distances based on shell caliber), and the spectator distance from the fallout zone ($D_{of}$).

The model used to perform the AAD calculation is illustrated in Figure 1. The effects of mortar placement and observer location on perceived burst size were demonstrated by varying $D_{fm}$ and $D_{of}$ for various shell sizes.

AAD is found by determining the solid angle (at the observer) encompassing the edges of the sphere. Since we are assuming spherical bursts, this analysis can be reduced to determining the angle between two lines originating at the observer, and tangent to the circle. Elevation is determined as a function of burst height, and distance to the observer.

The relevant trigonometric relations associated with this model are:

$$D_{ob} = \sqrt{\left(D_{of} + D_{fm}\right)^2 + H_{bur}^2}$$

$$A_{elev} = \tan^{-1} \frac{H_{bur}}{\left(D_{of} + D_{fm}\right)}$$

$$A_{aad} = 2\sin^{-1} \frac{R_{bur}}{D_{ob}}$$

The AAD of the shell burst is dependent not only on the observer's distance from the mortar, but also the height achieved by the shell. Large caliber shells, while having larger bursts, also attain greater heights. Near the fallout zone boundary, the larger $H_{bur}$ value is a significant factor affecting the perceived size.

The closer a burst is to the observer (as measured by $D_{ob}$), the smaller it needs to be to result in any given AAD. It also follows that bursts which are further away must be correspondingly larger to be perceived as equivalent.

The NFPA distances used for the mortar placements are shown in Table 1. These are derived from the NFPA guidelines for a fallout radius of 70 feet per inch (0.84 m/mm) of shell diameter.

The burst height and radius data used for this paper are shown in Table 2. The height and burst radius data have come from private communication[4] as well as previous publications by Kosankes and Schwertly[5] (height), and Kosankes[6] (radius). The fitted data published by Wharton[7] confirms that the burst height data used can be considered typical for these shell sizes.

**Table 1. NFPA Fallout Distances for Shell Sizes Used in This Paper.**

| Shell Size in inches (mm) | | NFPA Fallout Distance in feet | (m) |
|---|---|---|---|
| 3 | (76) | 210 | (64) |
| 4 | (103) | 280 | (85) |
| 5 | (127) | 350 | (107) |
| 6 | (152) | 420 | (128) |
| 8 | (203) | 560 | (171) |
| 10 | (254) | 700 | (213) |
| 12 | (305) | 840 | (256) |

**Table 2. Typical Values for Burst Height and Radius for Various Shell Sizes.**

| Shell Size inches | (mm) | Typical Burst Height ($H_{bur}$) feet | (m) | Typical Burst Radius ($R_{bur}$) feet | (m) |
|---|---|---|---|---|---|
| 3 | (76) | 400 | (122) | 130 | (40) |
| 4 | (103) | 560 | (171) | 170 | (52) |
| 5 | (127) | 700 | (213) | 210 | (64) |
| 6 | (152) | 785 | (239) | 250 | (76) |
| 8 | (203) | 950 | (290) | 410 | (125) |
| 10 | (254) | 1070 | (326) | 430 | (131) |
| 12 | (305) | 1175 | (358) | 450 | (137) |

Three shell launching configurations were used in the analysis. The first configuration, which is typical at hand-fired displays, is to have all shells grouped in the same general area, with the separation distance consistent with the largest shell in the group. In this case 8-inch shells are considered the largest that would be hand-fired. Figure 2 illustrates the burst patterns relative to the fallout zone boundary for this configuration as viewed from the side. The negative distances shown in Figures 2–4 indicate placement within the shooting area.

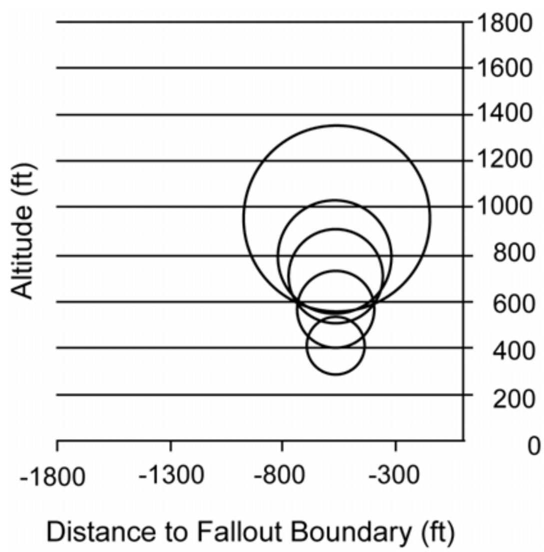



*Figure 2. Burst profile for configuration 1.*

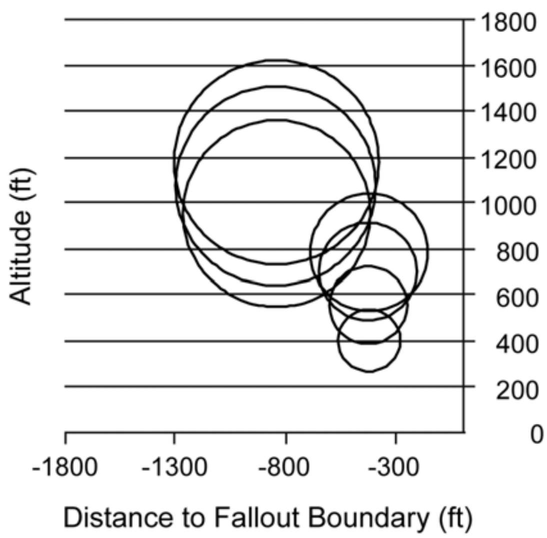



*Figure 3. Burst profile for configuration 2.*

The second configuration, typical at many displays, is to group all the small caliber shells up to 6-inch at one firing location 420 feet away, and all larger caliber shells at the location appropriate for the largest, in our example, 840 feet away for 12-inch shells. Figure 3 illustrates the profiles for this configuration.

In the third configuration, all mortars are simply distributed according to the NFPA guidelines. This results in 3-inch shells being placed 210 feet from the fallout zone boundary, 4-inch shells placed at 280 feet, and so on through 12-inch shells being placed 840 feet from the fallout zone boundary. Generally, arrangements like this would only be practical for electrically fired shows, where shoot crews would not need to be present among the various mortars. Figure 4 illustrates the burst profile for this configuration. All configurations assume the shells are launched from vertical mortars.

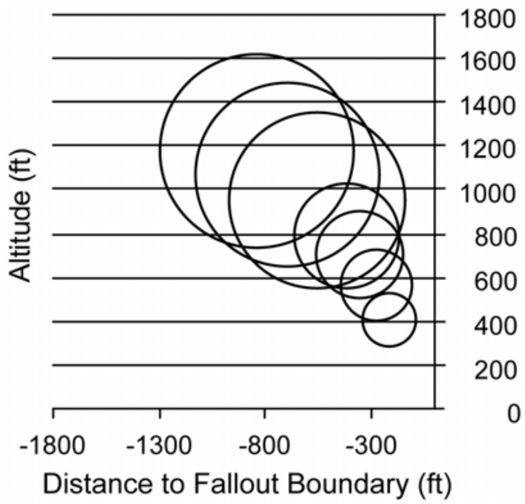



*Figure 4. Burst profile for configuration 3.*

A summary of the mortar placements for the three configurations relative to the fallout zone boundary is shown in Table 3.

**Table 3. Summary for the Three Shell Launching Configurations Used for the AAD Calculations.**

| | Mortar Distance to Fallout Boundary (ft) | | | | | | |
|---|---|---|---|---|---|---|---|
| | Shell Size (Inches) | | | | | | |
| | 3 | 4 | 5 | 6 | 8 | 10 | 12 |
| Config. 1 | 560 | 560 | 560 | 560 | 560 | — | — |
| Config. 2 | 420 | 420 | 420 | 420 | 840 | 840 | 840 |
| Config. 3 | 210 | 280 | 350 | 420 | 560 | 700 | 840 |

## Results

One of the main observations resulting from this investigation is that the largest caliber shells do not always result in the largest observed burst pattern, especially when the observer location is near the fallout zone boundary. This may be somewhat counter-intuitive, but at such close ranges, the effects of burst height and placement of the mortar relative to the observer are both significant contributors to the AAD.

Figure 5 shows the apparent angular diameters calculated for four shell sizes, fired at their NFPA distance from the fallout zone boundary. The diagram illustrates AAD results from the fallout zone boundary through 1 mile (1610 m) away. Even though the 12-inch shell has the largest actual burst radius of all the shells used (see Table 2), it does not have the largest apparent burst size near the fallout zone (compared to an 8-inch shell for example). This is due to the fact that a 12-inch shell needs to be fired further away from the spectators than an 8-inch shell, and it achieves a greater altitude before bursting. Both of these effects contribute to a significantly greater $D_{ob}$, resulting in a reduced AAD. Near the fallout zone boundary, there is virtually no observed AAD difference between a 3-inch shell burst and a 5-inch shell burst, and both of these are only within a few degrees of the observed 12-inch AAD. As observer distance in increased, the sizes of the shell bursts become more distinct.

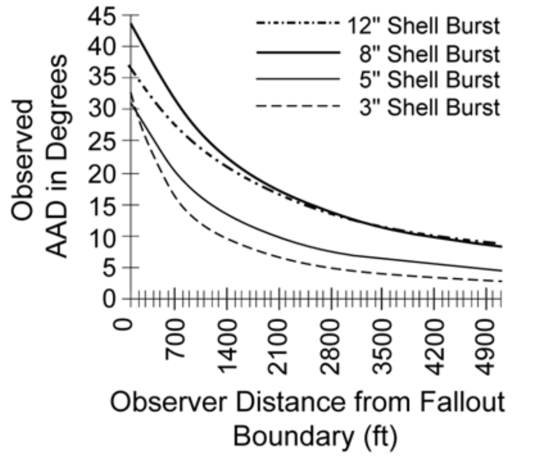



*Figure 5. AAD drop-off as a function of observer distance from the fallout zone (mortars placed per configuration 3).*

The AAD of the burst was not the only physical parameter that varied with observer distance. The elevation of the burst also changed. Elevation is the angle from the horizon to the center of the burst. An elevation of 0 degrees means that the center of the burst occurred on the horizon (as in a ground break), and an elevation of 90 degrees means that the burst would break directly overhead (as if one were standing directly under the burst).

The following figures illustrate the elevation and azimuth profile for each burst. Elevation is an absolute measurement from the horizon, and the azimuth is relative to the center of the burst.

The burst profile for an observer standing at the fallout zone boundary, with the mortars placed per configuration 1, is shown in Figure 6. In this example, the burst coverage appears as expected, with the smaller shells having a smaller burst diameter and appearing lower in the sky.

Figure 7 shows the burst profile for an observer standing in the same location, but seeing shells fired from mortars placed per configuration 2. The smaller shells have an AAD approaching the larger ones, and also appear to be higher in the sky.

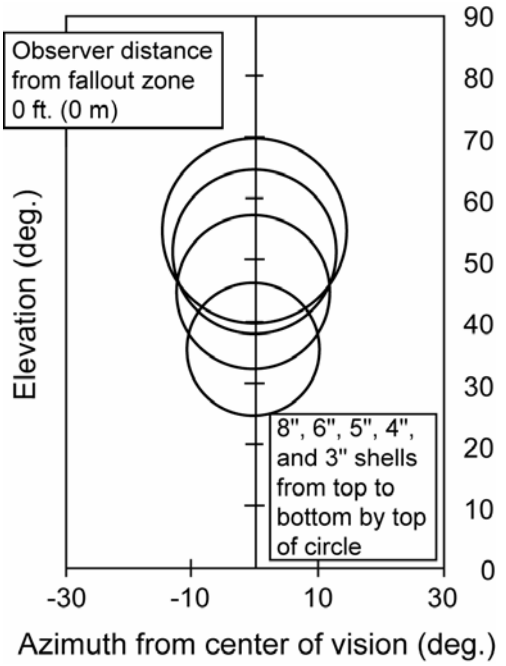



*Figure 6. Burst coverage for Configuration 1 viewed 0 feet from the fallout zone boundary.*

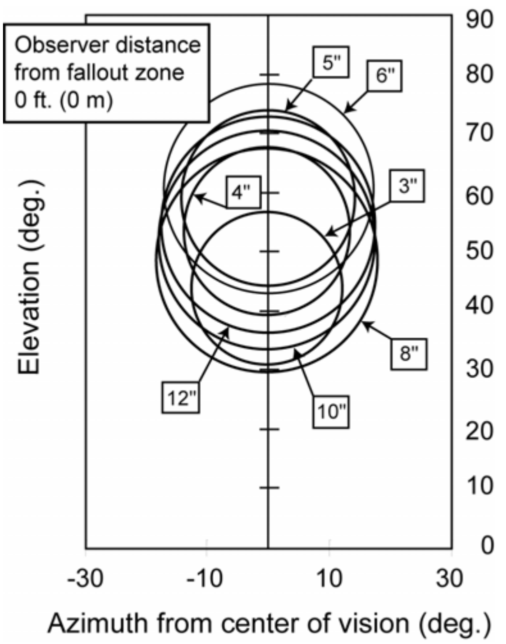



*Figure 7. Burst coverage for configuration 2 viewed 0 feet from the fallout zone boundary.*

Finally, the burst profile for configuration 3 is shown in Figure 8. Here there are significant differences in the observed bursts versus the first configuration.

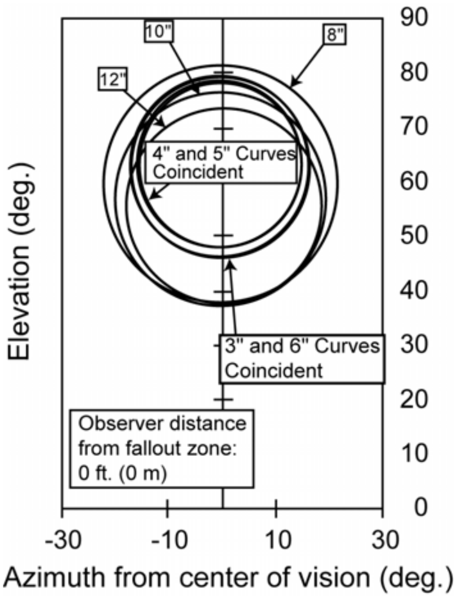



*Figure 8. Burst coverage for Configuration 3 viewed 0 feet from the fallout zone boundary.*

When viewed from the fallout boundary, it is virtually impossible to distinguish the burst sizes of the 3-inch shells and 6-inch shells. The same effect is also apparent with the 4-inch and 5-inch shells. Another interesting result is that the 8-inch shell burst appears larger than either the 10-inch or 12-inch shell bursts.

The 12-inch burst with an AAD of 36.3° is only 3.7° larger than the 6-inch burst AAD of 32.6°. It also appears smaller than the 8-inch burst having an AAD of 43.7°.

It is, therefore, easy to see how observers anticipating the large break of a 12-inch shell might be disappointed when viewing the display from near the fallout zone boundary. What may not be appreciated up close is the visual effect at greater distances. Figures 9 and 10 show the burst profiles for configuration 3 at distances of 500 and 2000 feet from the fallout zone boundary. As observer distance is increased, the actual differences in burst radius and height become more apparent.

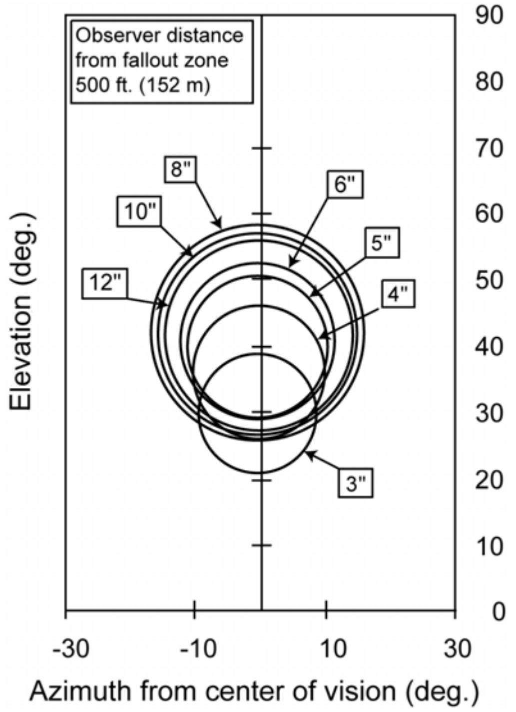



*Figure 9. Burst coverage for configuration 3 viewed 500 feet from the fallout zone boundary.*

When viewed from 2000 feet and beyond, all configurations have essentially the same appearance, because the observer's distance from the fallout zone ($D_{of}$) dominates the distance from the observer to the shell burst ($D_{ob}$) and therefore the observed burst size.

A summary of the elevations and apparent angular diameters for the three configurations is shown in Table 4. For each configuration (for $D_{of}$ values of 0, 500, and 2000 feet), both the calculated AAD and elevation is shown.

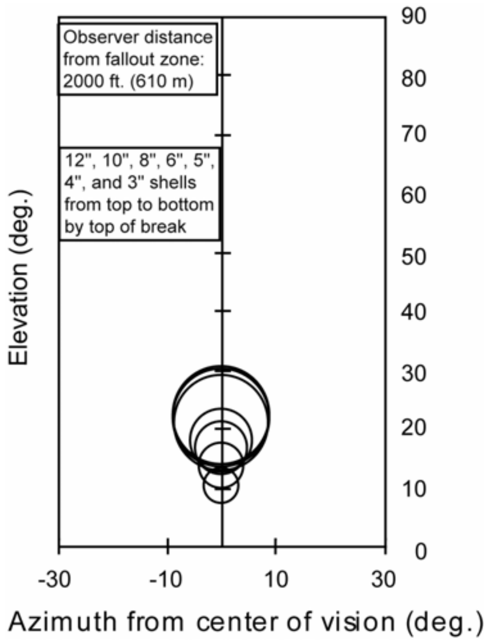



*Figure 10. Burst coverage for configuration 3 viewed 2000 feet from the fallout zone boundary.*

The greatest effects of varied mortar placement can be observed within 500 feet of the fallout zone boundary. Some inversion (larger actual bursts having smaller perceived size) of the AAD values for the larger shells can be seen at observer distances of 500 feet and below, especially for the NFPA–based mortar placement in configuration 3.

## Conclusion

Understanding perspective when designing shoot site layouts can lead to greater control of what the spectators see, and may contribute to more efficient use of materials. For example, if one were providing a display for a small party, where the spectators can be considered grouped rather narrowly along the fallout zone boundary, there may be no quantitative advantage to shooting 5-inch and 6-inch shells versus 3-inch and 4-inch shells, as they will appear nearly the same size when viewed. Such consideration may result in reduced display cost, or an increased number of shells to be presented.

In another possible extrapolation, if the majority of the spectators are to be within 2000 feet of the fallout zone boundary, there may be little advantage to presenting anything larger than an 8-inch shell. Unless many observers are a great distance away (or one wishes to present the display to an entire metro area) the addition of the larger effects may not be fully appreciated.

To achieve the largest overall vertical spread in the sky, one should collocate all mortars at the further firing positions (such as 560 feet away, as in configuration 1) and use a variety of shell sizes. Locating all mortars at their respective NFPA distances maximizes the perceived size of each burst, but also puts them at nearly the same elevation. The resulting effect can be bursts “stepping on” other bursts, with little variety in elevation. Collocating all mortars would result in a more “layered” effect.

Clearly, the visual impact of aerial shells depends not only on the size of the shell itself, but also observer location and mortar placement. In some situations, smaller, less costly shells can produce bursts of equal or larger perceived diameter than larger shells fired at the greater distances necessitated by the NFPA table of distances.

If these parameters are carefully considered, an impressive and effective display can be staged even if large fallout zones are not available or budget is limited. Careful attention to the effects of perspective can also help the pyrotechnician plan the shooting site to maximize the artistic effects he or she is trying to achieve.

While a purely quantitative analysis might indicate that smaller shells fired closer to the fallout zone boundary might be a more effective way to maximize the perceived burst size, there are other reasons to use large shells. There is a clear qualitative difference between large and small shells of similar effect. There is something majestic and beautiful about the slow-motion opening and complexity presented by large shells, which smaller shells cannot match.

**Table 4. Summary of AAD and Elevation Values for All Three Mortar Placement Scenarios at 0, 500, and 2000 feet from the Fallout Zone Boundary.**

| | | Shell Sizes, AAD, and Elevation (all values in degrees) | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 3-inch | | 4-inch | | 5-inch | | 6-inch | | 8-inch | | 10-inch | | 12-inch | |
| | Feet | AAD | Elev | AAD | Elev | AAD | Elev | AAD | Elev | AAD | Elev | AAD | Elev | AAD | Elev |
| Config. 1 | 0 | 21.8 | 35.5 | 24.8 | 45.0 | 27.1 | 51.3 | 30.1 | 54.5 | 43.7 | 59.5 | | | | |
| | 500 | 13.2 | 20.7 | 16.3 | 27.8 | 19.0 | 33.4 | 21.9 | 36.5 | 33.5 | 41.9 | | | | |
| | 2000 | 5.8 | 8.9 | 7.4 | 12.3 | 9.1 | 15.3 | 10.7 | 17.0 | 17.3 | 20.4 | | | | |
| Config. 2 | 0 | 25.9 | 43.6 | 28.1 | 53.1 | 29.8 | 59.0 | 32.6 | 61.9 | 37.7 | 48.5 | 36.9 | 51.9 | 36.3 | 54.4 |
| | 500 | 14.9 | 23.5 | 18.2 | 31.3 | 20.9 | 37.3 | 23.9 | 40.5 | 28.9 | 35.3 | 29.0 | 38.6 | 29.3 | 41.2 |
| | 2000 | 6.1 | 9.4 | 7.8 | 13.0 | 9.6 | 16.1 | 11.3 | 18.0 | 15.7 | 18.5 | 16.3 | 20.6 | 16.8 | 22.5 |
| Config. 3 | 0 | 33.4 | 62.3 | 31.5 | 63.4 | 31.1 | 63.4 | 32.6 | 61.9 | 43.7 | 59.5 | 39.3 | 56.8 | 36.3 | 54.4 |
| | 500 | 18.4 | 29.4 | 20.4 | 35.7 | 22.0 | 39.5 | 23.9 | 40.5 | 33.5 | 41.9 | 31.0 | 41.7 | 29.3 | 41.2 |
| | 2000 | 6.6 | 10.3 | 8.3 | 13.8 | 9.8 | 16.6 | 11.3 | 18.0 | 17.3 | 20.4 | 17.0 | 21.6 | 16.8 | 22.5 |

Where economics, shooting location, and audience distribution allow, ten-inch, twelve-inch, and larger shells certainly have their place. However if they are to be fully appreciated by spectators and customers, the closest viewing positions are not necessarily the best.

Overall, a pyrotechnician is responsible for applying all applicable factors when planning a display to ensure satisfaction, safety, and value to the customer. Manufacturers should also be able to make use of perspective to optimize their product for maximum visual performance.

While characterizing the visual perspective effects of fireworks displays is only one aspect of the art and science of pyrotechnics, a practical understanding of it can be an invaluable tool to the fireworks manufacturer, show planner, and display operator.

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