# Observations on the Heights Attained by Spherical Firework Shells 

Roland K. Wharton<br>Health and Safety Laboratory, Health and Safety Executive<br>Harpur Hill, Buxton, Derbyshire, SK17 9JN, United Kingdom


#### Abstract

This paper examines previously published information on the heights attained by spherical firework shells and proposes some empirical relationships that enable rough estimates to be made of the height attained from knowledge of either the mass or the diameter of the shell.


Keywords: shell height, shell mass, lift charge mass, shell diameter

## Introduction

Shimizu ${ }^{[1]}$ has discussed factors that can be important in determining the height attained by shells launched from mortars, and has presented some data illustrating the relationships between height and the mass of both the shell and its lift charge.

Related work from Contestabile and coworkers ${ }^{[2]}$ has also provided some information, and studies of this general type have helped to identify a number of variables that can be im-
portant in determining shell height. For example, the fit of the shell in the tube, the composition and grain size of the lift charge, the mass of the lift charge, the type of shell, its density, the mass of the shell, and the length of the mortar are all factors that can exert an influence.

Kosanke and Kosanke have also reported work that has examined the role of mortar tilt angle and wind speed in determining the height attained by shells. ${ }^{[3]}$ Additionally, they have examined the lateral drift of shells from their predicted paths caused by the tumbling motion in flight. ${ }^{[4]}$

As well as the factors affecting shell height identified above, the burning time of the fusing arrangement (delay time) will also be important in determining the height at which the burst occurs.

A recent paper ${ }^{[5]}$ reported noise measurements for a range of fireworks and also contained some information, see Table 1, relating to the heights that can be attained by various spherical shells. These results were examined initially by plotting the height of burst of each shell against

Table 1. Heights Attained by Various Spherical Fireworks Shells.

|  | Gross <br> Mass, $M_{1}$ | Lift Charge <br> Mass, $M_{2}$ | Height <br> $(\mathrm{m})$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(\mathrm{g})$ | $(\mathrm{g})$ |  |  |  |

both the gross mass $\left(\mathrm{M}_{1}\right)$ and the mass of the lift charge $\left(\mathrm{M}_{2}\right)$. The forms of dependence found prompted the examination of literature data presented in this paper.

Much of the published data on shell heights relates to experimental shells which may not always fit generally accepted design criteria. These are included in this paper because of the lack of any other detailed information, but it is possible that some of the findings may have to be amended when more results for normal shells become available.

## Discussion

Fulcanelli ${ }^{[6]}$ and literature from Westech ${ }^{[7]}$ suggest that linear relationships can link $\mathrm{M}_{2}$ with $M_{1}$. This would require that the ratio of $M_{2} / M_{1}$ is constant, which is not the case for the shells listed in Table 1 and was not found by Contestabile ${ }^{[2]}$ or on examination of Shimizu's data. ${ }^{[1]}$

Similarly, the dependence of height attained on the ratio of propelled mass $\left(\mathrm{M}_{1}-\mathrm{M}_{2}\right)$ to
propelling mass $\left(\mathrm{M}_{2}\right)$ for shells from two independent sources ${ }^{[1,5]}$ indicates that different relationships can apply.

Shimizu ${ }^{[1]}$ has reported different dependences on $\mathrm{M}_{2}$ of the height attained by 120 and 150 mm shells, and, although the basis for these observations was not clear, it was felt that an important factor could be that not all the energy produced by the lift charge is used to drive the shell from the mortar. It is possible that the observed differences could relate to the relative magnitude of the losses incurred in propelling gases from the mortar tube. ${ }^{[1]}$

The quantity of lift charge used in a shell is related to its mass and size ${ }^{[8]}$ for ballistic reasons and also because the magnitude of the effects from larger shells creates an additional requirement for them to function at progressively greater altitudes. The height attained by shells, if optimum fuse burning times apply and no other considerations predominate, would therefore be expected to be related to the energy input at launch. Figure 1 shows that if launch noise measured at 25 m is used as a rough measure of


Figure 1. The dependence of height of burst on launch noise for shells with diameters $75-125 \mathrm{~mm}$.
energy input to the shell, then a relationship of this type holds for those shells of 75,100 and 125 mm diameter in Table 1 for which launch sound pressure level measurements are available. ${ }^{[5]}$

However, the limited data in Table 1 indicate that the height of burst decreases with $\mathrm{M}_{2}$ for each size of shell, as was reported by Shimizu ${ }^{[1]}$ for 150 mm shells containing more than 60 g of lift charge. This may reflect the different design approaches of the manufacturers of specific fireworks since a number of groupings are discernible (e.g., white shells are characterized by high $\mathrm{M}_{2}$ and low heights of burst for each shell size).

It is possible to examine these results in a wider context by comparison with additional published work. Figure 2 summarises Shimizu's data, including some work with large diameter shells (ref. 1, Table 35). Although different modes of construction may have been employed for the shells and a number of other variables could be important in influencing performance, the general trend indicates an increase in burst height with $\mathrm{M}_{2}$ up to a height of about 400 m ,
when considerations relating to visibility by the audience may start to predominate as a design issue. The form of Figure 2 is in agreement with that reported by Shimizu ${ }^{[1]}$ (for 150 mm diameter shells) and by Contestabile ${ }^{[2]}$ (for 140 mm diameter shells with $\mathrm{M}_{1}=1361 \mathrm{~g}$ ); in both studies the height attained by the shells was seen to decrease at high values of $\mathrm{M}_{2}$.

Shimizu has also examined the dependence of the height attained by shells on their mass, and attempted to obtain an indication of the optimum shell mass to give a maximum height. From the published results ${ }^{[1]}$ for 120 mm shells, it is apparent that in order to attain a maximum height, the mass of the shell must be greater than 550 g . For 150 mm shells, Shimizu concludes that a maximum height of about 280 m would be obtained with shells having a mass of 1.2 kg .

The heights reached by those aerial fireworks in Table 1 for which the gross mass was known were plotted on the same graph as results from the literature, primarily those published by Shimizu. The resulting dependence, Figure 3, indicates that, even with some variation in the per-


Figure 2. The relationship between height of burst and the mass of lift charge for a range of shell sizes.
formance of individual shells of a particular type, height increases with $M_{1}$ and an optimum height of about 400 m is reached as $\mathrm{M}_{1}$ approaches 5 kg . Although the range of shell masses in the study by Contestabile and coworkers ${ }^{[2]}$ was limited, data calculated for the heights of the (mainly dummy) shells were found to be evenly distributed about the line summarising the general form of the dependence in Figure 3. As a result of sample variability, Figure 3 contains insufficient data to substantiate previous proposals ${ }^{[1,2]}$ that for each shell size there is a certain mass that will give a maximum height.

Details of the determination of shell burst heights from acoustic measurements have been presented in the literature ${ }^{[9]}$ and work has recently been published ${ }^{[10]}$ reporting the application of an electronic method. ${ }^{[11]}$ The results for the mean height obtained using this technique are plotted with Shimizu's data and the mean values from reference 5 in Figure 4.

It has previously been reported ${ }^{[11]}$ that shells burst at heights of 100 feet per inch of shell diameter, and this relationship was subsequently modified by Kosanke, Schwertley and Kosanke ${ }^{[10]}$ to incorporate a 150 feet correction factor for shells having diameters of less than 12 inches ( 305 mm ).

Published data indicate substantial variations in the performance of shells of the same type, e.g., in 8 trials with 255 mm shells ${ }^{[10]}$ the highest burst height was 422 m and the lowest 206 m , and different modes of construction between shell types will also exert an influence. Nevertheless, Figure 4 suggests that there is a strong correlation between mean burst height and shell size. The dependence shown passes through the origin and yields the approximate relationship of 1.4 m of height per mm of shell diameter.

The general relationship in Figure 4 for spherical shells may also apply to cylindrical shells since a height of 105 m has been reported by Kosanke and Kosanke for a 100 mm cylindrical shell. ${ }^{[12]}$


Figure 3. The dependence of height of burst on shell mass.


Figure 4. The dependence of the mean height of burst of shells on diameter.

## Conclusion

Even though many variables (e.g., mode of construction, fuse burning time, mortar diameter) can affect the performance of firework shells, examination of literature data for the heights attained has indicated that common empirical relationships may link the height of burst with the shell diameter and the gross mass for a range of spherical shells.

## References

1) T. Shimizu, Fireworks from a Physical Standpoint, Part III, Pyrotechnica Publications, Austin, TX, 1985.
2) E. Contestabile, R. A. Augsten, T. R. Craig, and J. G. McIndoe, Proc. $13^{\text {th }}$ IPS, Grand Junction, CO, USA, 1988, p 141.
3) K. L. Kosanke and B. J. Kosanke, Pyrotechnica XIV, 1992, p 46.
4) K. L. Kosanke and B. J. Kosanke, Proc. 17th IPS, Beijing, China, 1991, p 96.
5) R. K. Wharton and H. J. Slater, Pyrotechnica XVI, 1995.
6) A. Fulcanelli, Pyrotechnica $I X, 1984$, p 7.
7) Westech, "Basic Aerial Shells", Westech Corporation, 1970.
8) B. J. Kosanke and K. L. Kosanke, American Fireworks News, No. 99, 1989, p 1.
9) J. G. Taylor, Pyrotechnica X, 1985, p 36.
10) K. L. Kosanke, L. A. Schwertley and B. J. Kosanke, PGI Bulletin, No. 68, 1990, p 12.
11) K. L. Kosanke, PGI Bulletin, No. 64, 1989, p 14.
12) K. L. Kosanke and B. J. Kosanke, American Fireworks News, No. 157, 1994, p 1.
