Chapter 8 — Examples of Designing Chrysanthemum Shells

8.1 Comparison of Calculated and Experimental Values

The accuracy of the empirical equation for the initial velocity of a star was examined in Section 6.5.5. The accuracy of flight velocity can be examined by the probability deviations of the constant *n*' and log v_o. Therefore, the author only compared the results obtained from the empirical equation with the results of experiment.

Example: From the Data Given Below, Calculations of the Flight Distance of Stars.

Calculation Number	1	2	3
Construction	$D_4B_8PM_4$	$D_5B_{12}HW_8$	$D_5G_{12}PW_8$
Size of Shell	4 sun	5 sun	6 sun
Star	B_8	B_{12}	G_{12}
T_1	0.72 s	1.60 s	2.92 s
T_2	0.60 s	0.60 s	0.60 s
δ_1	1620 kg/m^3	1620 kg/m^3	1640 kg/m^3
δ_2	1650 kg/m^3	1650 kg/m^3	1650 kg/m^3
R_1	0.00435 m	0.00600 m	0.00600 m
R_2	0.00250 m	0.00250 m	0.00250 m
m	0.00055kg	0.00147 kg	0.00156 kg
Bursting charge	**P**	**H**	**P**
A	1.93	5.99	1.93
f	7.12×10^4 m	7.36×10^4 m	7.12×10^4 m
$\hat{w}$	0.116 kg	0.234 kg	0.256 kg
Pasting paper	Kraft	Japanese paper	Japanese paper
a	2.33	1.00	1.00
n	4	8	8
Δ	0.30	0.30	0.30

Note: **S** was omitted because it is the weakest of the three and not useful except in some special cases.

(a) Calculation of Initial Velocity

Equations:

$$\bar{V} = K \cdot f_1 \cdot f_2 \cdot f_3$$

where

$$K = 5.91$$

$$f_1 = \left(f\varpi\right)^{0.450}$$

$$f_2 = \left(\frac{m}{\sigma}\right)^{-0.377}$$

$$f_3 = A^{-0.352} e^{-\frac{13.42}{anA}}$$

(Table 7 in the Appendix)

	Calculation Number		
	1	2	3
f	7.12×10^4	7.36×10^4	7.12×10^4
$\hat{w}$	0.116	0.234	0.256
$f\hat{w}$	0.826×10.4	1.722×10^4	1.823×10.4
f_1	57.91	80.57	82.68
σ	$3.14\times(0.00435)^2$	$3.14\times(0.00600)^2$	$3.14\times(0.00610)^2$
m	0.00055	0.00147	0.00156
m/σ	9.26	13.00	13.35
f_2	0.432	0.380	0.377
A	1.93	5.99	1.93
an	9.32	8.00	8.00
f_3 (Table 7 in Appendix)	0.376	0.403	0.333

Multiplying K by the values of f_1, f_2, and f_3 we have:

Calculated V	55.6	72.9	61.3
Experimental V	47.3	71.4	76.1
Calc. V/Exp. V	1.18	1.02	0.81

The above table shows the coincidence of the values of initial velocity between the calculated and experimental.

(b) Calculation of the Flight Distance of the Star in the Horizontal Direction

During the time from the start to the end of the burning of the blue or green zone, excluding the bright flash from the burst charge, the flight distance was calculated. For this process, the final velocity v_1 was first calculated.

Equations.

$$v_1 = VF_v\left(A_{s,}b_1,\tau_1^{'}\right)$$

$$A_{s_1} = 0.774V^{0.93}\frac{T_1'}{R_1\delta_1}$$

$$b_1 = -\left(1-\frac{\delta_2}{\delta_1}\right)\left(\frac{R_2}{R_1}\right)^3$$

$$T_1' = \frac{T_1}{1-R_2/R_1}$$

$$\tau_1^{'} = \frac{t_1}{T_1'}$$

	Calculation Number		
	1	2	3
T'_1	1.694	2.744	5.008
R_1	0.00435	0.00600	0.00610
δ_1	1620	1620	1640
$V^{0.93}$	41.97	53.99	45.96
A_{s_1}	7.80	11.79	17.82
b	0.0034	0.0013	0.0004
τ_1'	0.425	0.583	0.583

Assuming $b = 0$ and using the values of A_{s_1} and τ_1' in the table of F_x, we have:

F_x	0.249	0.242	0.193
V	55.6	72.9	61.3
T'_1	1.694	2.744	5.008

Multiplying the above three values mutually, we have:

Calculated x_1	23.4 m	48.5 m	59.2 m

The experimental values were:

Experimental x_1	28.0 m	52.1 m	63.1 m
Star Number	112	371	542

The value of T_1 was measured experimentally. The result shows that the calculated values of the flight distance were 3.6 to 4.6 m smaller than the experimental values.

8.2 An Example of Designing a Chrysanthemum Shell

In this paper only an example of designing calculations was introduced. In practical cases more important factors need to be considered: for example, ease of manufacture, efficiency of explosive, etc.

Example: A Flower of 80 m Horizontal Radius, Amber to Blue Color with a Horizontal Flight Distance of 40 m Blue and 40 m Amber.

(a) Calculation of the Second Burning Period (Blue Colored).

From the given condition:

$$x_2 = 40.0 = v_1 T_2' F\left(A_{s_2}, b_2, \tau_2'\right) \qquad \text{(i)}$$

In this equation, v_1 is the final velocity of the star at the end of the first period and at the same time it is the initial velocity of the second period. x_2 is the distance traveled in the second period. The symbols for the first period are noted by a subscript of 1 and those of the second period are noted with a subscript 2. Therefore:

$$T_2' = T_2$$

$$A_{s_2} = 0.774 v_1^{0.93} \frac{T_2}{R_2 \delta_2} = 0.774 v_1^{0.93} \frac{1}{w_2 \delta_2}$$

From experiment (Tables 5 and 16):

$$w_2 = 0.00195 \text{ m/s}$$

$$\delta_2 = 1620 \text{ kg/m}^3$$

Therefore

$$A_{s_2} = 0.2450 v_1^{0.93} \qquad \text{(ii)}$$

On the other hand the value of $v_{0.95}$ is calculated as:

$$v_{0.95} = v_1 F_v\left(A_{s_2}, 0, 0.95\right) \qquad \text{(iii)}$$

From (ii) several v_1 values are obtained, and from (iii) $v_{0.95}$ values are obtained as shown in the following Table.

A_{s_2}	v_1	$v_{0.95}$
1	4.54	1.85
2	9.56	2.41
3	14.78	2.68
4	20.14	2.84
5	25.59	2.94
6	31.13	3.02
7	36.74	3.05
8	42.42	3.10
9	48.14	3.13
10	53.91	3.18
11	59.72	3.17
12	65.59	3.21
13	71.48	3.22
14	77.41	3.25
15	83.37	3.25
16	89.35	3.22

The relationship between the data is illustrated in Figure 46.

Figure 46 shows that when the values of A_{s_2} exceed 9, the final velocity of the star does not increase as much. Therefore, a value of less than 9 may be desirable. The larger the value of $v_{0.95}$, the less the star is affected by gravity. For this requirement, the value of A_{s_2} should be as large as possible under the condition $A_{s_2} < 9$. If we use $A_{s_2} = 9$, then we have:

$$v_1 = 48.14,\ v_{0.95} = 3.13$$

Then the radius of curvature of the trajectory of the star from equation 72' is:

$$p\cos\theta = -\frac{v^2}{g} = -\frac{(3.13)^2}{9.80} = -1.0 \text{ m}$$

The value of 1.0 m is too small. This means that the final velocity of the star is too small. However, if we possibly have stars with a large density or large final velocity, the problem may be successfully solved. From (i) we have:

$$T_2 = \frac{40}{v_1 F_x(9,0,1)}$$

From the above Table we have:

$$F_x(9,0,1) = 0.318$$

Therefore,

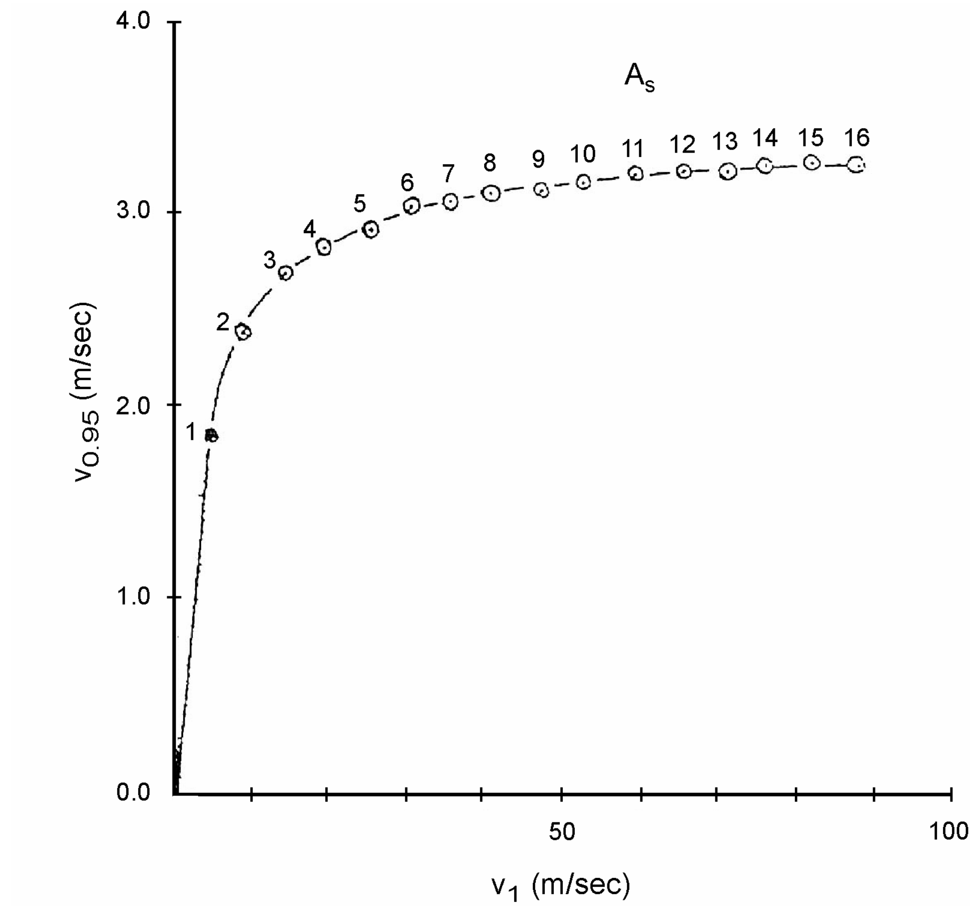

Figure 46. The values that are calculated from v_1 and the function F_v.

$$T_2 = \frac{40}{48.14 \times 0.318} = 2.61 \text{ s}$$

From this value, we can determine the radius of the second layer at the beginning of the blue:

$$\begin{aligned} R_2 &= w_2 T_2 \\ &= 0.00195 \times 2.61 \\ &= 0.00509 \text{ m} \\ &= 5.09 \text{ mm} \end{aligned}$$

(b) Calculation of the First Burning Period (Amber Color)

From an experiment:

$$\delta_1 = 1480 \text{ kg/m}^3$$

$$\begin{aligned} w_1 &= \frac{12.26 - 4.92}{2} \Big/ 2.17 - 0.60 \\ &= 0.00234 \text{ m/s } \{4.3.1\ (4) \text{ or } (6)\} \end{aligned}$$

Connecting this period with the second period:

$$VF_v\left(A_{s_1}, 0, \tau_1'\right) = v_1 = 48.14 \qquad \text{(iv)}$$

$$VT_1'F_x\left(A_{s_1}, 0, \tau_1'\right) = x_1 = 40 \qquad \text{(v)}$$

And:

$$\begin{aligned} A_{s_1} &= 0.774V^{0.93} F \frac{1}{w_1 \delta_1} \\ &= 0.774V^{0.93} \frac{1}{0.00234 \times 1480} \\ &= 0.2235V^{0.93} \end{aligned}$$

From this equation the relationship of A_{s_1} and V is obtained as follows:

A_{s_1}	23	24	25	26	27
V (m/sec)	145.6	152.4	159.3	166.1	173.1

In this case the final value of τ' is unknown. It is assumed with a proper value of τ' that the

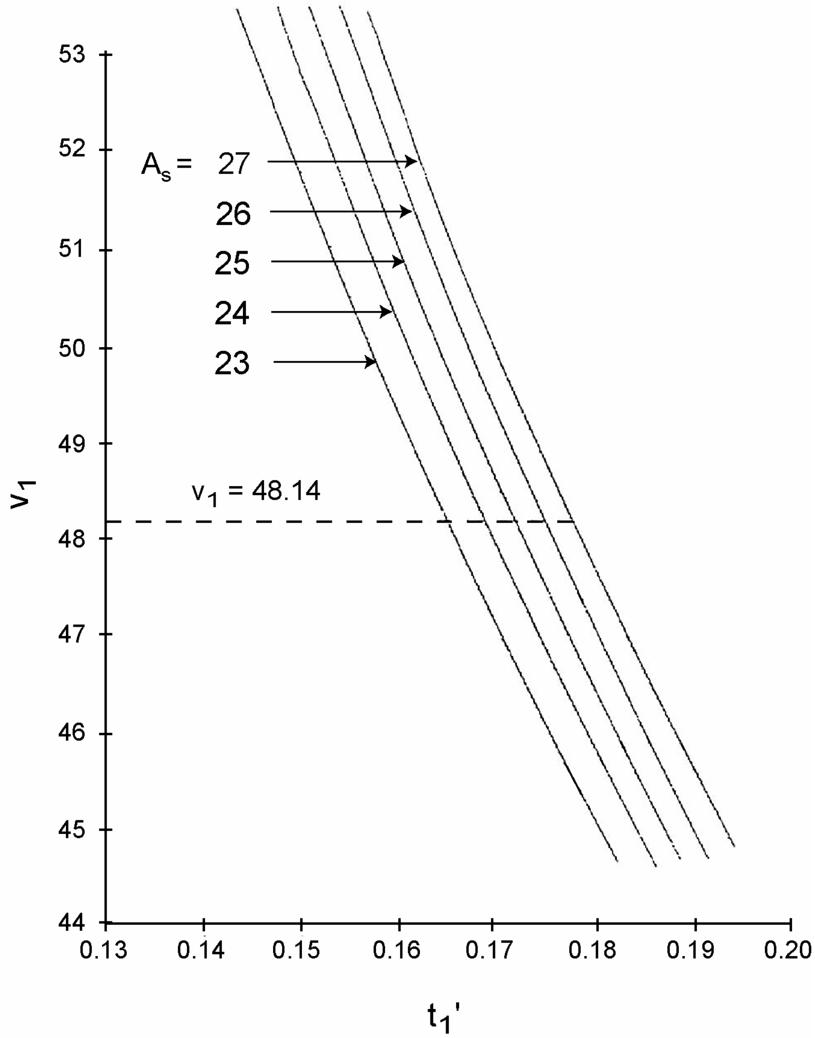

Figure 47. The relation of v_1 and τ_1'.

$$v_1 = VF_v\left(A_{s_1}, 0, \tau_1'\right)$$

$\tau_1 \backslash A_{s_1}$	23	24	25	26	27
0.12	60.13	61.26	62.45	63.62	64.70
0.14	54.16	55.17	56.23	56.97	57.96
0.16	49.21	50.14	50.82	51.66	52.25
0.18	44.99	45.72	46.36	47.01	47.58
0.20	41.35	41.91	42.38	43.02	43.42
0.22	38.15	38.56	39.05	37.53	39.96

The above data are plotted as a graph in Figure 47, and the final velocity of v_1 = 48.14 m/sec is recorded as a dashed horizontal line, and the points of intersection give the value of τ'_1, these denote the relations of values A_{s_1} and τ_1':

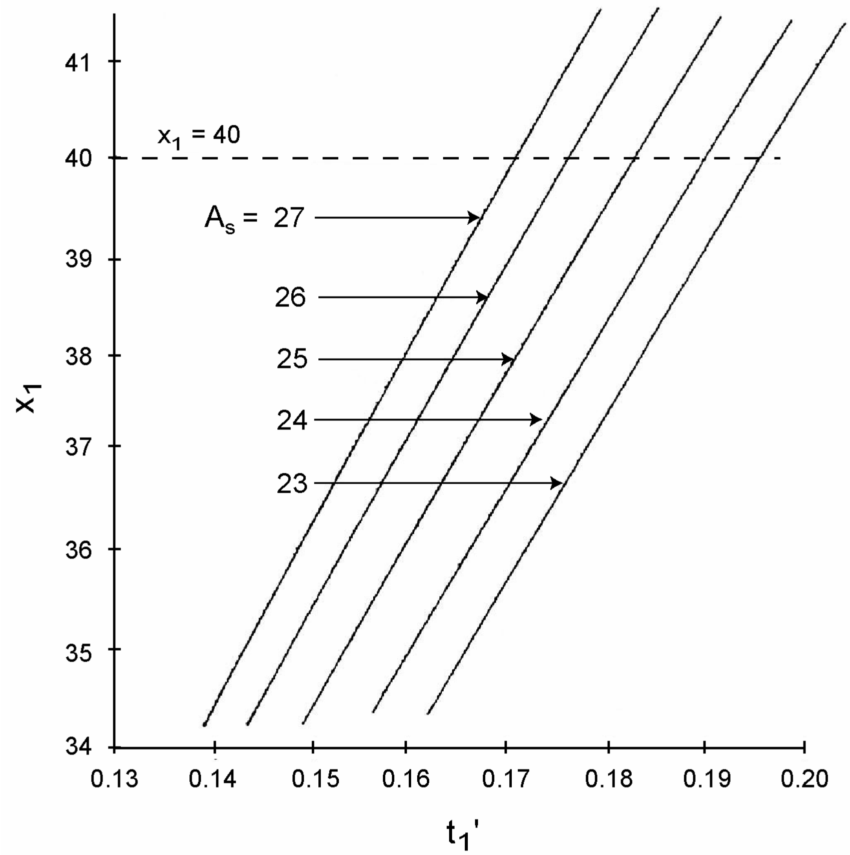

Figure 48. The relationship of x_1 and τ_1'.

(I)

A_{s_1}	23	24	25	26	27
τ_1'	0.1647	0.1685	0.1720	0.1750	0.1775

And then,

$$\tau_1' = 1 - R_2/R_1$$

From this relation:

$$R_1 = \frac{R_2}{1-\tau_1'} = \frac{0.00509}{1-\tau_1'}$$

On the other hand:

$$T_1' = \frac{T_1}{1-R_2/R_1} = \frac{R_1}{w_1} = \frac{R_1}{0.00234} = 427.4R_1$$

Combining the above two equations:

$$T_1' = \frac{2.175}{1-\tau_1'}$$

From this, the values of T_1' are obtained as follows:

(I)

τ_1'	0.12	0.14	0.16	0.18	0.20	0.22
T_1'	2.471	2.530	2.558	2.654	2.719	2.788

Using these values and the value of V, the travel distance x_1 during the first period is calculated as follows:

$$x_1 = VT_1'F_x\left(A_s, 0, \tau_1'\right)$$

$\tau_1 \backslash A_{s_2}$	23	24	25	26	27
0.12	27.35	28.24	29.13	29.97	30.79
0.14	30.56	31.63	32.64	33.62	34.59
0.16	34.29	35.09	36.28	36.96	38.07
0.18	37.47	38.43	39.73	40.55	41.77
0.20	40.76	41.85	42.88	44.27	45.16
0.22	44.25	45.03	46.20	47.23	48.23

These values are plotted in Figure 48.

In Figure 48, the condition was given that x = 40 m. It is shown in Figure 48 as a horizontally dashed line, and when we read the value of t_1' at the points of intersection with the A_{s_2} curves, we have:

(II)

A_{s_2}	23	24	25	26	27
τ_1'	0.1950	0.1890	0.1820	0.1755	0.1700

From the data of (I) and (II) we have the two curves on the graph in Figure 49.

The point of intersection denotes the values of A_{s_1} and V that satisfy the conditions given before:

$$A_{s_2} = 26.12$$
$$\tau'_1 = 0.175$$

For the value of A_{s_2} we have:

$$V = 167.0 \text{ m/s}$$

Therefore, we have:

$$R_1 = \frac{R_2}{1-\tau_1'} = \frac{0.00509}{1-0.175} = 0.00617 \text{ m} = 6.17 \text{ mm}$$

Therefore the diameter of the star is:

$$d = 0.00617 \times 2 = 0.01234 \text{ m} = 12.3 \text{ mm}$$

Then we have:

$$T_1' = \frac{R_1 - R_2}{w_1} = \frac{0.00617 - 0.00509}{0.00234} = 0.46 \text{ s}$$

The solution to the first period is completed with the above calculations.

The burning time of the star in total:

$$T = T_1 + T_2 = 0.46 + 2.61 = 3.07 \text{ s}$$

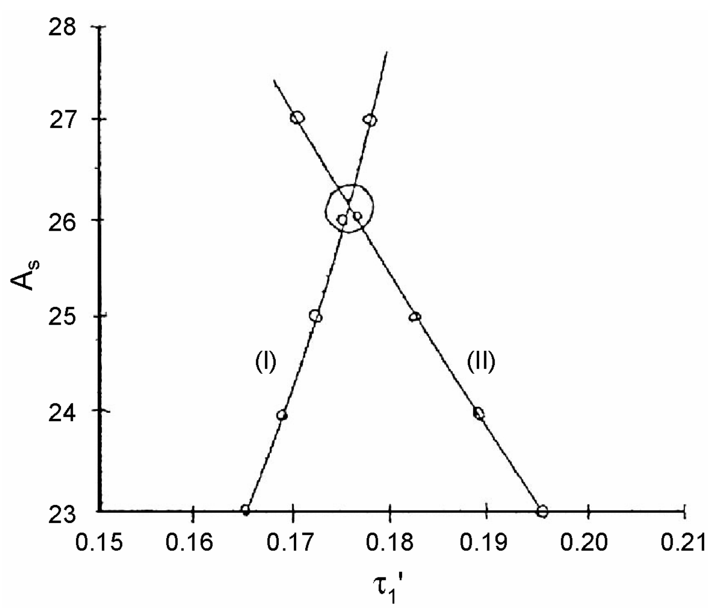

Figure 49. Graph for deciding the value of A_{s_2} and τ'_1.

The mass of the single star:

$$m = \frac{4}{3}\pi\left\{\left(R_1^3 - R_2^3\right)\delta_1 + R_2^3\delta_2\right\}$$
$$= \frac{4}{3}\times 3.14\times\left[\begin{array}{l}\left\{\left(0.00617\right)^3 - \left(0.00509\right)^3\right\} \\ \times 1480 + \left(0.00509\right)^3 \times 1620\end{array}\right]$$
$$= 0.00153 \text{ kg}$$
$$= 1.53 \text{ g}$$

$$\frac{m}{\sigma} = \frac{0.00153}{3.14\times\left(0.00617\right)^2}$$
$$= 12.80 \text{ kg/m}^2$$

In the above calculation it was assumed $b_1 = 0$. This is checked:

$$b_1 = -\left(1 - \frac{\delta_2}{\delta_1}\right)\left(\frac{R_2}{R_1}\right)^3$$
$$= -\left(1 - \frac{1620}{1480}\right)\left(\frac{0.00509}{0.00617}\right)^3$$
$$= 0.053$$

This value does not disturb the condition $b = 0$.

The above calculation concerns the design of a star flying horizontally, but the falling height of the trajectory is as follows, ignoring air resistance:

$$y = \frac{1}{2}gT^2$$
$$= \frac{1}{2}\times 9.80\times\left(3.07\right)^2$$
$$= 46.2 \text{ m}$$

The Design of the Shell

First an assumption is made that there is no difference between ring star and full star shells with regard to the design conditions. As described earlier, the initial velocity of the full star is somewhat greater than that of the ring star. However, this does not affect our manufacturing process.

The initial velocity from equation 108 is:

$$V = K\cdot f_1\cdot f_2\cdot f_3$$
$$= 167.0 \text{ m/s}$$

From this relation we have the value of f_1:

$$f_1 = \frac{V}{K\cdot f_2\cdot f_3} \qquad \text{(vii)}$$
$$= \frac{167.0}{K\cdot f_2\cdot f_3}$$

On the other hand:

$$f_2 = \left(p/\sigma\right)^{-0.377}$$
$$= \left(12.80\right)^{-0.377}$$
$$= 0.382$$

When we use the bursting charge **P**, the characteristic values are:

$$f = 7.12\times 10^4,\ A = 1.93$$

For shell pasting, when we use Kraft paper (cement bag) for 6 layers:

$$an = 2.33\times 6$$
$$= 13.98$$

From the values of **an** and A, the values of f_3 are calculated (or from Table 7, value f_3)

$$f_3 = A^{-0.352}e^{-\frac{13.42}{anA}}$$
$$= \left(1.93\right)^{-0.352}e^{-\frac{13.42}{13.98\times 1.93}}$$
$$= 0.483$$

Therefore, from (vii)

$$f_1 = \frac{167.0}{5.91\times 0.382\times 0.483}$$
$$= 153.1$$
$$= \left(f\varpi\right)^{0.450}$$

From above, the value of $f\varpi$ is

$$f\varpi = 71710 \text{ kg}\cdot\text{m}$$

When we divide $f\varpi$ by f = 71200 m, we have 1.007 kg.

The quantity of cottonseeds ϖ', which fixes the bursting charge ϖ, assumes ϖ/ϖ' = 16:

$$\varpi = w/1.600$$
$$= 0.629 \text{ kg}$$
$$\varpi' = 0.629 \text{ kg}$$

Therefore the total mass of the burst charge is:

$$W = \varpi + \varpi'$$
$$= 1.007 + 0.629$$
$$= 1.636$$
$$= 1.600 \text{ kg}$$

The apparent specific gravity of the bursting charge is 0.556 and the volume of the charge is:

$$v = \frac{1.636}{0.556}$$
$$= 2.942 \text{ liters}$$

When the volume is spherical, the radius is:

$$R = \left(\frac{3}{4} \cdot \frac{1}{\pi} \times 2.942 \right)^{\frac{1}{3}}$$
$$= 0.889 \text{ dm}$$
$$= 89 \text{ mm}$$

Including the radius of the star:

$$R + d = 101 \text{ mm}$$

Therefore the inside diameter of the shell is d_i = 101 × 2 = 202 mm. For the 7 sun (~8 inch) shell, the recommended number of stars should be:

$$n = 4.35 \frac{D}{d} \left(\frac{D}{d} - 2 \right)$$
$$= 4.35 \times \frac{187}{12} \left(\frac{187}{12} - 2 \right)$$
$$= 922 \text{ stars}$$

The efficiency of the bursting charge when reacted should be:

$$r = \frac{\text{total moving energy of stars}}{\text{energy of the burst charge}}$$
$$= \frac{1}{2} \cdot mV^2 n \bigg/ \frac{f \varpi g}{\gamma - 1}$$
$$= \frac{1}{2} 0.00153 \times (167.0)^2 \times 922 \bigg/ \frac{71710 \times 9.80}{1.25 - 1}$$
$$= 0.0070$$
$$= 0.7\%$$

where γ is the adiabatic thermal expansion coefficient of the burst charge gas. The gravitational acceleration g appears in the above equation because the force of explosives f was calculated on the basis of a force of 1 kg-weight × 1 meter, which is 9.80 joules. To convert (kg-weight × meters) into joules we must multiply by 9.80.