Chapter 7 — Equations for Practical Use in Designing Shells

7.1 The Characteristics of Stars and the Preparation of Tables of Star Ballistics

Equations 71 and 72 are written again:

$$v = v_0 \left\{ 1 - \left(\frac{A_s}{3} \right) \log \frac{(1-\tau')^3 + b}{1+b} \right\}^{-1.075} \quad (71)$$

$$x = v_0 T' \int_0^{\tau_1'} \left\{ 1 - \left(\frac{A_s}{3} \right) \log \frac{(1-\tau')^3 + b}{1+b} \right\}^{-1.075} d\tau' \quad (72)$$

When ballistic functions are set as follows:

$$F_v = F_v(A_s, \tau', b) = \left\{ 1 - \left(\frac{A_s}{3} \right) \log \frac{(1-\tau')^3 + b}{1+b} \right\}^{-1.075} \quad (101)$$

$$F_x = F_x(A_s, \tau', b) = \int_0^{\tau'} \left\{ 1 - \left(\frac{A_s}{3} \right) \log \frac{(1-\tau')^3 + b}{1+b} \right\}^{-1.075} d\tau' \quad (102)$$

equations 71 and 72 become:

$$v = v_0 F_v \quad (103)$$

$$x = v_0 T' F_x \quad (104)$$

The tables in the Appendix list the calculated values from equations 101 and 102 where $b = 0$ and $b = 0.5$.

7.2 Investigation of Star Ballistics in the Air from the Ballistic Tables

From equations 103 and 104 we have:

$$F_v = \frac{v}{v_0}, \; F_x = \frac{x}{v_0 T'}$$

Therefore, for a single colored star, the function F_v shows the ratio of the flight velocity to the initial velocity, and F_x shows the ratio of the actual flight distance to the imaginary flight distance with no air resistance. The larger the values of F_v and F_x are, the greater the movement of the star. Figures 41A and B and 42A and B show the values of the functions F_v and F_x, when b = 0.0 and b= 0.5.

The values of A_s are denoted as follows from equation 70' without the subscript 1 for R, T', and δ

$$A_s = 0.774 v_0^{0.93} \frac{1}{R/T'} \cdot \frac{1}{\delta}$$

In the above equation, *R/T'* means the burning velocity and δ means the density of the burning layer. Therefore, when the value of v_o is constant, the larger the burning velocity, the smaller the value of A_s, and the larger the specific density, the smaller the value of A_s. Figures 41 and 42 show that the smaller the value of A_s, the larger the values of F_v and F_x. Therefore, when the burning velocity *R/T'* and the specific density of the star are larger, then the star flies further.

The value of *b* is denoted from equation 67' as follows:

$$b = \left\{ \begin{array}{l} \ldots + \left(\frac{R_4}{R_1} \right)^3 \left(\frac{\delta_4}{\delta_1} - \frac{\delta_3}{\delta_1} \right) + \\ \left(\frac{R_3}{R_1} \right)^3 \left(\frac{\delta_3}{\delta_1} - \frac{\delta_2}{\delta_1} \right) + \left(\frac{R_2}{R_1} \right)^3 \left(\frac{\delta_2}{\delta_1} - 1 \right) \end{array} \right\}$$

If there is a relationship between the layers of a star:

$$\delta_1 < \delta_2 < \delta_3 < \ldots$$

the value of *b* becomes larger and larger as burning proceeds. Comparing $b = 0$ and $b = 0.5$ in the graphs in Figures 41 and 42, we see that the decrease in velocity of the star is less with $b = 0.5$ than with $b = 0.0$. Furthermore, the flight distance for b = 0 is greater than for $b = 0.5$. In

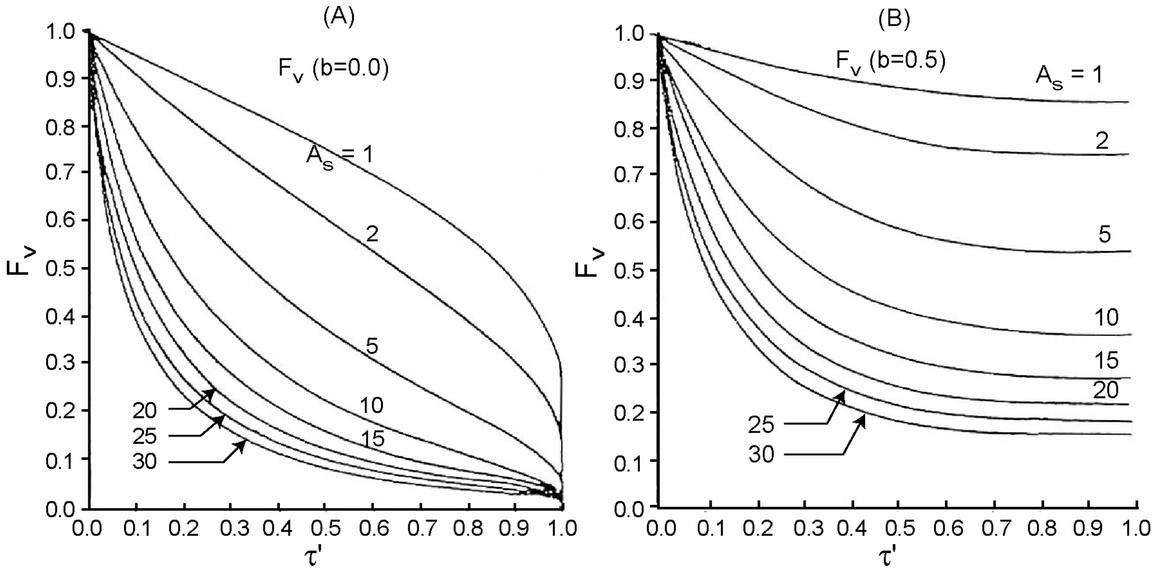

Figure 41. Velocity functions.

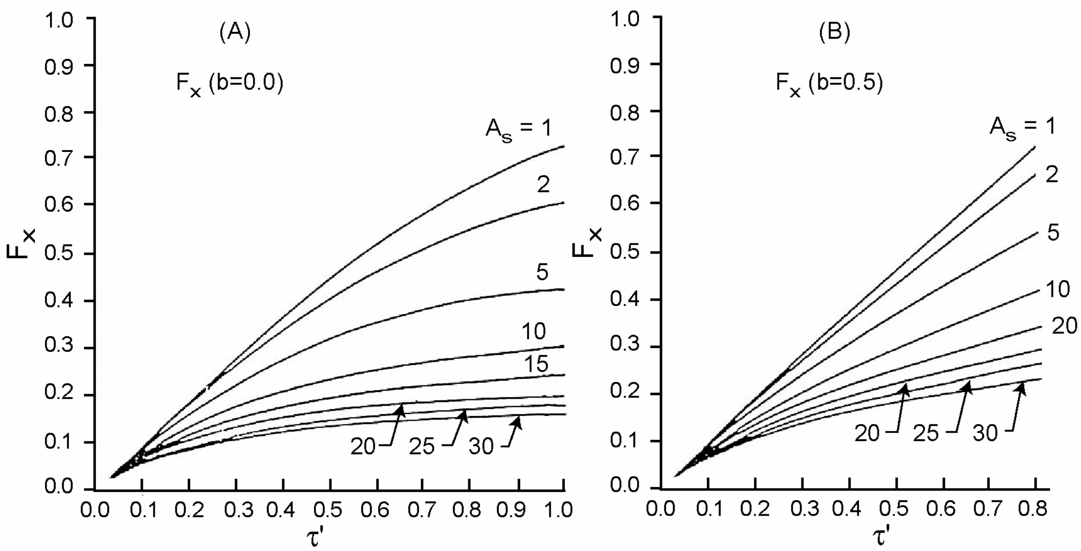

Figure 42. Distance functions.

other words, a star, of which the density becomes greater and greater, produces the best effect. Special attention should be paid to the condition when $b = 0$ and $F_v = 0$. However, when $b > 0$ under $\tau = 1$, F_v has a limited value.

7.3 The Method For Obtaining the Function $F_v(\tau', A_s, b)$ from $F_v(\tau', A_s, 0)$

The ballistic tables in the Appendix were made for $b = 0$ and $b = 0.5$. For other values of b, they do not correspond. However, with the following method, it is possible to obtain the data from $F_v(t', A_s, 0)$. The function of the problem is set as:

$$F_v(\tau_x', A_s, b)$$

It is equated with:

$$F_v(\tau_x', A_s, 0) = F_v(\tau', A_s, b) \qquad (105)$$

Both sides are substituted into equation 72 to give:

$$\left\{1-\left(\frac{A_S}{3}\right)\log\left(1-\tau_x^{'}\right)^3\right\}^{-1.075}$$
$$=\left\{1-\left(\frac{A_S}{3}\right)\log\frac{\left(1-\tau'\right)^3+b}{1+b}\right\}^{-1.075}$$

From this equation we have:

$$\log\left(1-\tau_x^{'}\right)^3=\log\frac{\left(1-\tau'\right)^3+b}{1+b}$$
$$\tau_x^{'}=1-\sqrt[3]{\frac{\left(1-\tau'\right)^3+b}{1+b}}=\tau\left(\tau',b\right) \quad (106)$$

Therefore, the value of $F_v(\tau',A_s,b)$ is equal to the value $F_v(\tau'_x,A_s,0)$, which is calculated with the value τ'_x. Therefore from the table for $F_v(\tau',A_s,0)$ we can obtain the values of $F_v(\tau',A_s,b)$. For this purpose a table of $F(\tau',b)$ was prepared. The values of $F_x(\tau,A_s,b)$ are simply not available. In this case it is necessary to calculate the function $F_v(\tau',A_s,b)$ and to integrate step by step.

7.4 An Examination of the Equations of Initial Velocity of Stars

From equation 100, setting three equations as:

$$f_1=\left(f\varpi\right)^{0.450}$$
$$f_2=\left(\frac{p}{\sigma}\right)^{-0.377} \quad (107)$$
$$f_3=A^{-0.352}e^{-\frac{13.42}{anA}}$$

we have

$$\overline{V}=5.91\times\left(1\pm0.084\right)f_1\cdot f_2\cdot f_3 \quad (108)$$

Here it should be noted that equations 100 and 108 are empirical and apply for loading densities in the range 0.27 ~ 0.33.

Special attention should be given to Figure 45 with the function f_3. When different types of paper are used with the same value of **an** in total, the value of f_3 moves along the same **an** curve. When the value **an** is relatively small, the curve f_3 does not show the maximum. However, when **an** shows a high value, the curve shows a maximum as **an** = 12 or 16. For example, when pasting with standard Japanese paper on a shell where **an** = 8 and A = 5.0, we have f_3 = 0.41. If the bursting charge is replaced with that which shows a slower burning rate (A = 1.5), the same value for f_3 as above can be obtained when **an** = 12. In this case, one should paste 12 layers per sun. When the pasting layers of paper increase to a large amount, the effect of the force on projecting stars by the bursting

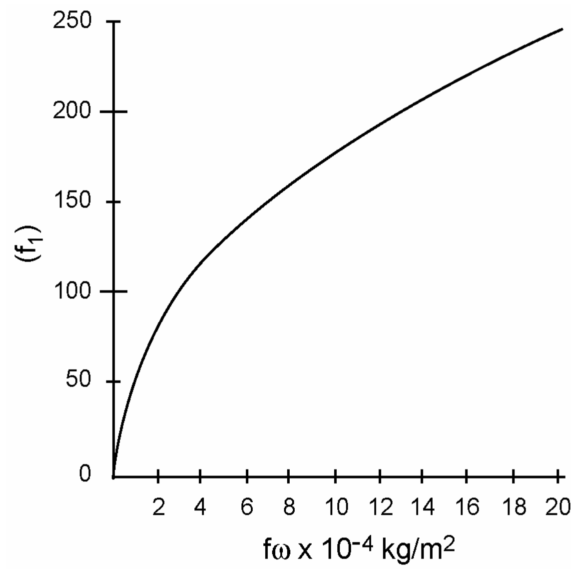

Figure 43. Function f_1 for the initial velocity of star.

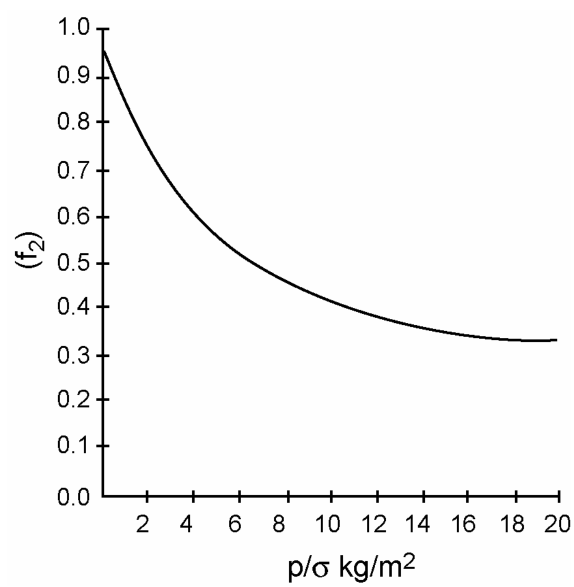

Figure 44. Function f_2 for initial velocity of star.

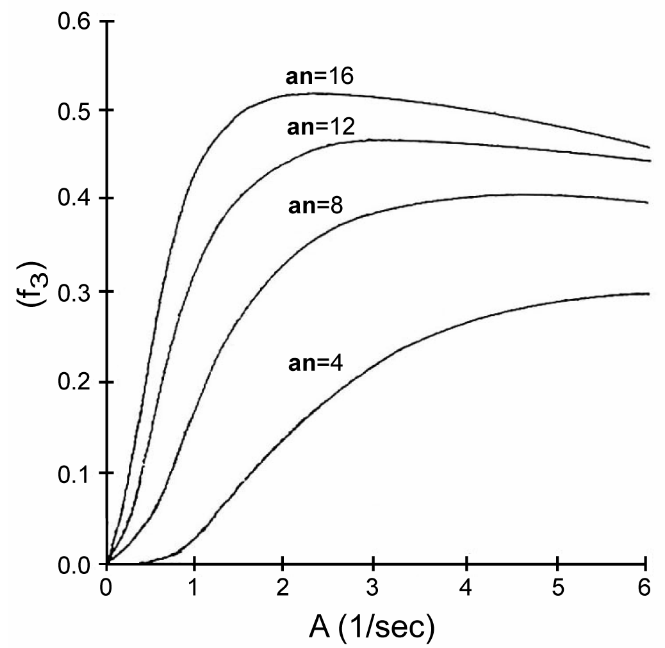

Figure 45. Function f_3 for initial velocity of star.

charge increases in a similar manner.

7.5 The Relationship between the Deviations of Initial Velocity $\frac{\Delta V}{V}$ and the Deviations of Flight Distances $\frac{\Delta x_1}{x_1}$

The flight distance of a star along the horizontal axis during the burning of the first layer of the star is shown in equation 104.

$$x_1 = VT_1'F_x\left(A_s, \tau', b_1\right)$$

If the construction of the star is unchanged and T'_1, τ', b_1 are constant, the equation is differentiated as:

$$\frac{dx_1}{x_1} = \frac{dV}{V} + \frac{dF_x}{F_x} \qquad (109)$$

On the other hand:

$$\frac{dF_x}{F_x} = \left(\frac{A_s}{F_x} \cdot \frac{\partial F_x}{\partial A_s}\right)\frac{dA_s}{A_s} \qquad (110)$$

If, as an approximation, the exponent of V is put as 1,

$$A_s = 0.774V \cdot \frac{1}{w_1\delta_1} \qquad (111)$$

If this is then differentiated by the logarithmic method, we have:

$$\frac{dA_s}{A_s} = \frac{dV}{V}$$

Introduce this into equation 110:

$$\frac{dF_x}{F_x} = \left(\frac{A_s}{F_x} \cdot \frac{\partial F_x}{\partial A_s}\right)\frac{dV}{V} \qquad (112)$$

Introducing equation 112 into equation 109:

$$\frac{dx_1}{x_1} = \frac{dV}{V}\left(1 + \frac{A_s}{F_x} \cdot \frac{\partial F_x}{\partial A_s}\right) \qquad (113)$$

In equation 113 we set:

$$\Sigma A_s = 1 + \frac{A_s}{F_x} \cdot \frac{\partial F_x}{\partial A_s} = \Sigma A_s\left(A_s, \tau', b\right) \qquad 114)$$

and when dx_1 and dV are replaced by the deviations Δx and ΔV, equation 113 becomes:

$$\frac{\Delta x_1}{x_1} = \Sigma A_s \frac{\Delta V}{V} \qquad (115)$$

Equation 115 is generalized as:

$$\frac{\Delta x_i}{x_i} = \Sigma A_s \frac{\Delta v_{0_i}}{v_{0_i}} \tag{115'}$$

Equations 115 and 115' are equations to denote the relationship of the deviation of the initial velocity and horizontal flight distance. The values of $\Sigma A_s = \left(1 + \frac{A_s}{F_x}\frac{\partial F_x}{\partial A_s}\right)$ are shown in the Appendix in Table 2.

Generally, as the value of A_s increases, the values of ΣA_s become smaller. This shows that as the value of A_s increases, small deviations in the value of the initial velocity do not have much affect upon the flight distances.