

Part I. A Theoretical Development of the Line-Reversal Method for Flame Temperature Measurement

ABSTRACT

A fireworks flame generally contains many solid or liquid particles, which cause a continuous spectrum. In order to apply the line-reversal method of temperature measurement to such flames, the author introduced a theoretical equation, which denotes the ratio of the intensity of the resonance lines to that of the neighboring part of the spectrum when a standard light beam is introduced into the spectro-scope through the flame.

This equation shows very clearly that as long as the flame does not contain so many particles that it prevents the standard light beam from permeating the flame, the line-reversal method is always effective.

Using this equation, the author proposes a method of measuring flame temperatures that are higher than the maximum brightness temperature of the standard light. The author applied this method to two examples of high-temperature fireworks flames of some magnesium powder compositions and obtained the temperatures of 3,159 and 3,214 K.

I. Introduction

There are many historical formulations for colored-flame firework compositions. However, they are fragmentary, and the author did not find any literature that described the mechanisms of colored-flame production. The purpose of this paper is to explain experimentally the color-producing mechanism and to have general principles for designing colored-flame compositions for practical uses.

Colored-flame compositions are divided into two types according to their color-producing mechanisms: one uses atomic spectra and molecular spectra from flames and the other uses continuous spectra from solid or liquid materials at high temperature. This paper concerns only the former; the latter may be examined in the future.

Initially it is necessary to measure flame temperatures to explain the color-producing mechanism. There are several flame temperature measuring methods;^[1] however, the author used the line-reversal method, using Na-D lines, which is the most reliable.

When a small quantity of a metal salt of sodium, lithium, thallium, etc., is added to a composition, it produces resonance lines in its flame spectrum. When one observes the spectrum of a standard light, which produces a continuous spectrum, through the sample flame using the device in Figure 1, the resonance lines appear dark when the black body temperature of the standard light (τ) is higher than that of the flame (T). On the other hand, when the black body temperature of the standard light is lower, the lines look bright. Therefore, to read the black body temperature on the scale of the standard light, when the lines disappear while regulating the intensity of the light, this value is equal to the flame temperature, which is defined by the radiation theory.

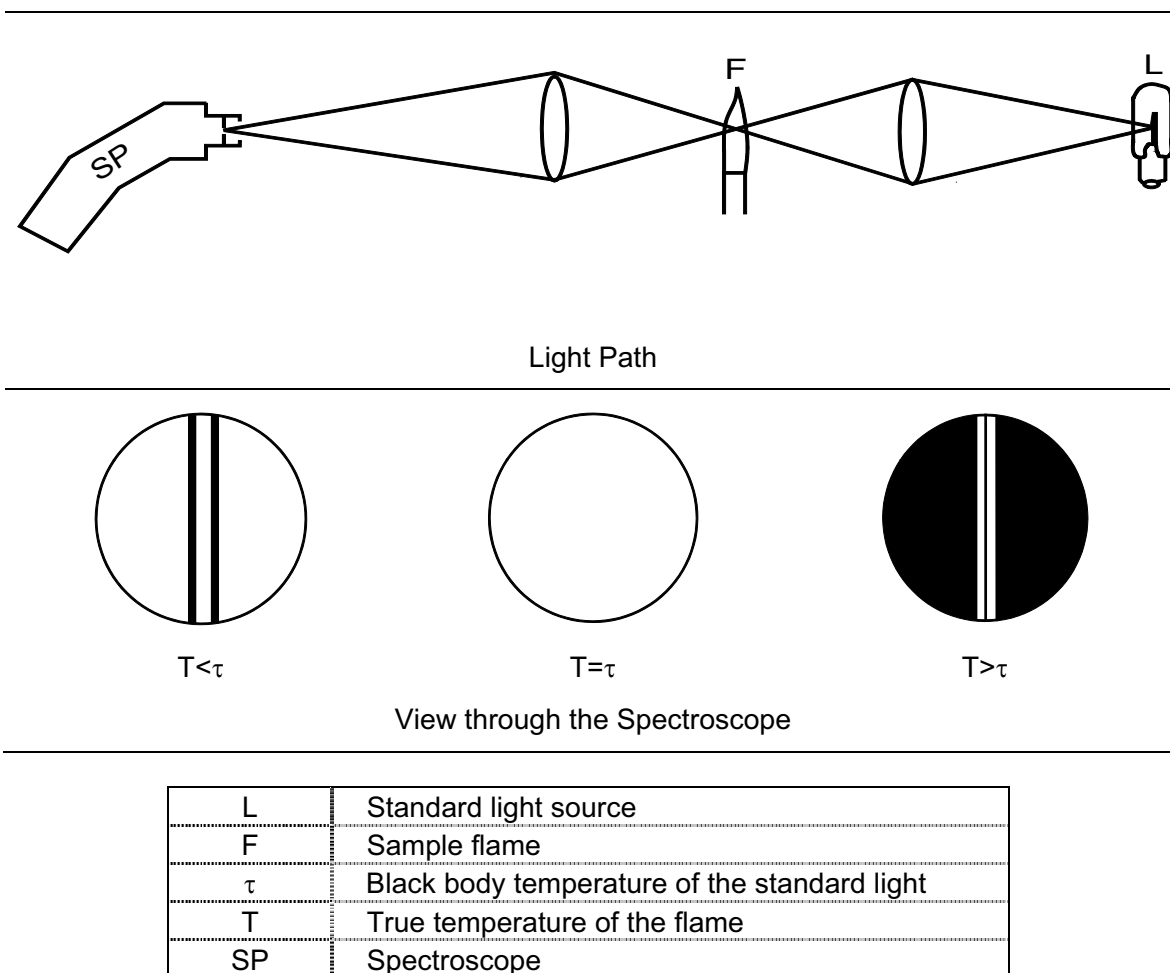


Figure 1. The principle of flame temperature measurement using the line-reversal method.

The following two problems need to be addressed before applying the line-reversal method to the measurement of firework flame temperatures.

(1) How does one obtain the theoretical solution when the flame contains solid or liquid particles?

(2) In general, by the ordinary line-reversal method, we cannot measure flame temperatures when they are higher than those of the standard light. How does one obtain flame temperatures that are greater than the maximum black body temperature of the standard light?

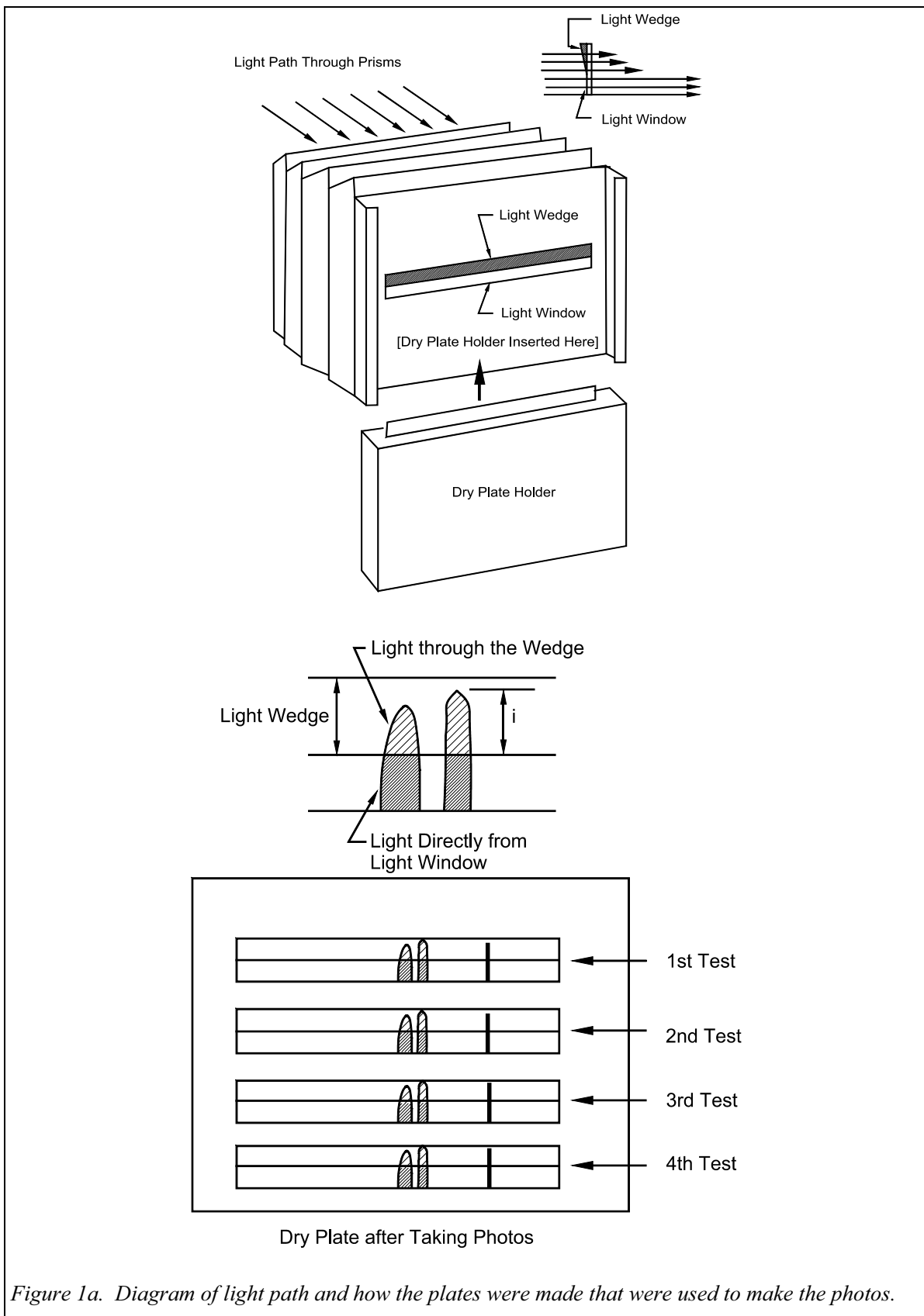


Figure 1a. Diagram of light path and how the plates were made that were used to make the photos.

II. Symbols Used in Equations

T	Temperature of flame in absolute units (K). The flame has a thickness, therefore, it denotes an average value along the path of the beams.
τ	Black body temperature of the standard light (K).
λ	Wavelength of light (cm).
$E(\lambda, T)$	Emissivity of an object at wavelength λ and true temperature T (with no subscript, denotes the black body).
$E(\lambda, \tau)$	Emissivity of an object at wavelength λ and black body temperature τ . It has the same value as the emissivity of black body when $T=\tau$.
$A(\lambda, T)$	Absorption coefficient of a body for light beams of wavelength λ .
$P(\lambda, T)$	Permeability coefficient of a body (equal to zero for an opaque body).
$R(\lambda, T)$	Reflection coefficient for light beams of wavelength λ .
r	Ratio of the light intensity of resonance lines to that of the neighboring background when observed with a spectroscope.
c_1	An emission constant ($11.906 \times 10^6 \text{ erg}\cdot\text{cm}^2\text{sec}^{-1}$).
c_2	An emission constant (1.4387 cm-deg).
The subscripted symbols are:	
o	Standard light source, it refers to the image of the standard light in the flame.
v	Refers to a flame that is assumed to consist completely of vapor.
s	Refers to a flame that is assumed to consist completely of solid (or liquid) particles.
F	Refers to the sample flame.
n	Refers to the part of the spectrum near the resonance lines.
D	Refers to the resonance lines (e.g., Na-D lines).
F_s	Refers to solid material in the flame.
F_v	Refers to vapor material in the flame.
c	Density of small solid or liquid particles in the flame.

For the symbol c it is assumed that the flame is a thin flat plate that is placed perpendicular to the axis of the beam and that the particles are distributed within the flame, but not outside of the flame.

III. Development of General Theoretical Equations for the Line-Reversal Method

The following conditions are necessary to use the line-reversal method.

- 1) Kirchhoff's law is applicable to the radiation of solid or liquid particles in the flame.
- 2) Kirchhoff's law is also applicable to the radiation of gaseous material in the flame.
- 3) There is no luminescence in the flame.

Condition (1) is empirically always correct. Condition (2) is true only when the radiation of the flame comes from the resonance lines (i.e.,

the absorbed light energy changes to the emission of light with the same wavelength). If there is a luminescence, condition (2) is not applicable.

When light beams of the standard light are introduced into the spectroscope through a sample flame, one observes an overlapping of three light images at the position of the wavelength λ_D of the resonance lines:

- (a) the image that comes from the continuous spectrum of the standard light, whose emissivity is denoted as: $P_F(\lambda_D, T, c) \cdot E(\lambda_D, \tau_o)$.
- (b) the image that comes from the resonance lines, whose emissivity is denoted as:
 $E_{F_s}(\lambda_D, T, c)$
- (c) the part of the image that is produced by solid or liquid particles and coincides with the wavelength of the resonance lines is denoted as: $E_{F_s}(\lambda_D, T, c)$.

The total sum of the emissivities is denoted as:

$$E_{F_v}(\lambda_D, T, c) + E_{F_s}(\lambda_D, T, c) + P_F(\lambda_D, T, c) \cdot E(\lambda_D, \tau_o) = E(\lambda_D, \tau'_o) \quad (1)$$

Namely, the observed light intensity is proportional to the value $E(\lambda_D, \tau'_o)$ of an imaginary light source.

In the part of the spectrum, outside the resonance lines, (b) is omitted. The sum of the emissivities is:

$$E_{F_s}(\lambda_n, T, c) + P_F(\lambda_n, T, c) \cdot E(\lambda_n, \tau''_o) = E(\lambda_D, \tau''_o) \quad (2)$$

Therefore, the intensity of the observed light is proportional to the value of $E(\lambda_D, \tau''_o)$ of another imaginary light source.

When the flame temporarily consists of only solid or liquid particles, condition (1) becomes as follows, introducing the Kirchhoff's law:

$$E_s(\lambda_D, T) = A_s(\lambda_D, T) \cdot E(\lambda_D, T) \quad (3)$$

$$E_s(\lambda_n, T) = A_s(\lambda_n, T) \cdot E(\lambda_n, T) \quad (4)$$

In the case of the sample flame, the density of the solid or liquid particles in the flame is not

as large as it is with the above formulas. Therefore the sample emission must be realized by multiplying some corrective coefficient to equations 3 and 4. Also the coefficient must be zero when the density of the particles is zero and must be one when the density is highest. Therefore, the coefficient has a relation to the permeability of the flame for light beams of wavelength λ . Temporarily we will use a value $(1 - P_{F_\lambda})$ [any form of the coefficient does not have influence upon the last effect]. See equations 10 and 12. From equations 3 and 4:

$$E_{F_s}(\lambda_D, T, c) = (1 - P_{F_\lambda}) \cdot E_s(\lambda_D, T) = (1 - P_{F_\lambda}) \cdot A_s(\lambda_D, T) \cdot E(\lambda_D, T) \quad (5)$$

$$E_{F_s}(\lambda_n, T, c) = (1 - P_{F_\lambda}) \cdot E_s(\lambda_n, T) = (1 - P_{F_\lambda}) \cdot A_s(\lambda_n, T) \cdot E(\lambda_n, T) \quad (6)$$

Secondly, when the flame consists of only gas (it is a type of transparent gas), it follows, applying equation 2:

$$E_v(\lambda_D, T) = A_v(\lambda_D, T) \cdot E(\lambda_D, T) \quad (7)$$

Normally the flame contains solid or liquid particles, therefore, from the same consideration as above, equation 7 is modified as:

$$E_{F_v}(\lambda_D, T, c) = P_{F_\lambda} \cdot E_v(\lambda_D, T) = P_{F_\lambda} \cdot A_s(\lambda_D, T) \cdot E(\lambda_D, T) \quad (8)$$

When equations 5 and 8 are introduced into equation 1, we have:

$$A_F(\lambda_D, T, c) \cdot E(\lambda_D, T) + P_F(\lambda_D, T, c) \cdot E(\lambda_D, \tau_o) = E(\lambda_D, \tau_o) \quad (9)$$

Here,

$$A_F(\lambda_D, T, c) = (1 - P_{F_\lambda}) \cdot A_s(\lambda_D, T) + P_{F_\lambda} \cdot A_v(\lambda_D, T) \quad (10)$$

denotes the absorption coefficient of the sample flame at the resonance lines. In the same way, when equation 6 is introduced into equation 2, we have:

$$A_F(\lambda_n, T, c) \cdot E(\lambda_n, T) + P_F(\lambda_n, T, c) \cdot E(\lambda_n, \tau_o) = E(\lambda_D, \tau_o) \quad (11)$$

Here,

$$A_F(\lambda_n, T, c) = (1 - P_{F_\lambda}) \cdot A_s(\lambda_n, T) \quad (12)$$

denotes the absorption coefficient of the sample flame against the light of wavelength λ_n .

The relationship of the absorption coefficient, permeability and reflection coefficient is physically denoted as:

$$A_F + P_F + R_F = 1 \quad (13)$$

This relationship is introduced into equations 9 and 11, and we have:

$$A_F(\lambda_D, T, c) \{E(\lambda_D, T) - E(\lambda_D, \tau_o)\} + \{1 - R_F(\lambda_D)\} \cdot E(\lambda_D, \tau_o) = E(\lambda_D, \tau'_o) \quad (14)$$

$$A_F(\lambda_n, T, c) \{E(\lambda_n, T) - E(\lambda_n, \tau_o)\} + \{1 - R_F(\lambda_n)\} \cdot E(\lambda_n, \tau_o) = E(\lambda_n, \tau''_o) \quad (15)$$

In these equations it seems that the reflection coefficient R_F is only relative to the solid particles and not to the gaseous materials.^[2] When the wavelengths λ_D and λ_n approach each other, we can have:

$$\left. \begin{aligned} E(\lambda_D, T) &\doteq E(\lambda_n, T) \\ E(\lambda_D, \tau_o) &\doteq E(\lambda_n, \tau_o) \\ R_F(\lambda_D) &\doteq R_F(\lambda_n) \end{aligned} \right\} \quad (16)$$

The operation, using the line-reversal method, compares the brightness of the resonance lines and the near part on the spectrum with each other. Therefore, the equation of comparison obtained by using equation 16 is:

$$r \doteq \frac{E(\lambda_D, \tau'_o)}{E(\lambda_n, \tau''_o)} = \frac{A_F(\lambda_D, T, c) \left\{ \frac{E(\lambda_D, T)}{E(\lambda_D, \tau_o)} - 1 \right\} + \{1 - R_F(\lambda_D)\}}{A_F(\lambda_n, T, c) \left\{ \frac{E(\lambda_n, T)}{E(\lambda_n, \tau_o)} - 1 \right\} + \{1 - R_F(\lambda_n)\}} \quad (17)$$

Rearranging the above equation, and introducing a newly defined function $F(r)$, we have:

$$\frac{E(\lambda_D, T)}{E(\lambda_D, \tau_o)} = 1 + \frac{\{1 - R_F(\lambda_D)\}(r - 1)}{A_F(\lambda_D, T, c) - r A_F(\lambda_n, T, c)} \equiv F(r) \quad (18)$$

From another source of Wien's radiant equation we have:

$$E(\lambda_D, T) = \frac{c_1}{\lambda^5} e^{-c_2/\lambda_D T} \quad (19)$$

$$E(\lambda_D, \tau_o) = \frac{c_1}{\lambda^5} e^{-c_2/\lambda_D \tau_o} \quad (20)$$

The ratio of the two equations is:

$$F(r) = e^{-\frac{c_2}{\lambda_D} \left(\frac{1}{T} - \frac{1}{\tau_o} \right)} \quad (21)$$

Changing the form, we have:

$$T = \frac{1}{\frac{1}{\tau_o} - 2.303 \frac{\lambda_D}{c_2} \log F(r)} \quad (22)$$

Equation 22 is the common theoretical equation for the measurement of flame temperatures using line-reversal method when the flame contains solid or liquid particles.

IV. Examination of the Common Equation When Flame Temperatures Are Lower Than the Black Body Temperature of the Standard Light (Method 1)

When the flame temperature is lower than the black body temperature of the standard light, it is possible to regulate the temperature of the standard light so that the intensity of the resonance lines becomes the same as that of the near part of the spectrum (brightness regulation). In this case $r = 1$ following equation 17. When this is introduced to equation 18, we have $F(r) = 1$. Since $r = 1$, then

$$E(\lambda_D T) = E(\lambda_D, \tau_o)$$

and from equations 19 and 20 we have:

$$T = \tau_o$$

Therefore, the flame temperature equals the black body temperature of the standard light source.

By the above reason, the line-reversal method is applicable whether or not the flame contains solid or liquid particles.

However, how is it when the particle density is very large? In such a case the value of P_{F_λ} decreases to zero ($P_{F_\lambda} \doteq 0$) and from equations 10 and 12 we have:

$$\begin{aligned} A_F(\lambda_D, T, c) &\doteq A_s(\lambda_D, T) \\ A_F(\lambda_n, T, c) &\doteq A_s(\lambda_n, T) \doteq A_s(\lambda_D, T) \end{aligned}$$

From equation 17 we have $r \doteq 1$. In this situation the errors in the operation of the brightness adjustment increase. Therefore, this method is not good when the particle density is very large.

V. Plan To Measure Flame Temperatures When the Temperature of the Flame Is Higher Than the Black Body Temperature of the Standard Light (Method 2)

For flame temperatures that are higher than the black body temperature of the standard light, it is necessary to develop another method because it is impossible to regulate the temperature of the standard light as high as the temperatures of the flame.

Deforming equation 18 we have:

$$F(r) = 1 + \frac{(r-1)}{\frac{A_F(\lambda_D, T, c)}{1 - R_F(\lambda_D)} - r \frac{A_F(\lambda_n, T, c)}{1 - R_F(\lambda_D)}} \quad (23)$$

Then the following variables are set:

$$x = \frac{A_F(\lambda_D, T, c)}{1 - R_F(\lambda_D)} \quad (24)$$

$$y = \frac{A_F(\lambda_n, T, c)}{1 - R_F(\lambda_D)} \quad (25)$$

where x or y denotes the ratio of the absorbed light energy of the resonance lines or that of the standard light to the light energy that is not reflected. Introducing this equation into equation 23, we have:

$$F(r) = 1 + \frac{(r-1)}{x - ry} \quad (26)$$

This is introduced into equation 21, and we have:

$$e^{-\frac{c_2}{\lambda_D} \left(\frac{1}{T} - \frac{1}{\tau_o} \right)} = 1 + \frac{r-1}{x - ry} \quad (27)$$

In this equation the values of x , y and T do not seem to vary as the black body temperature of the standard light is changed. Therefore, the value of τ_o is changed in three steps, as $\tau_{o_1}, \tau_{o_2}, \tau_{o_3}$, and when the values of r_1, r_2 , and r_3 are measured experimentally, we have three simultaneous equations:

$$e^{-\frac{c_2}{\lambda_D} \left(\frac{1}{T} - \frac{1}{\tau_{o1}} \right)} = 1 + \frac{r_1 - 1}{x - r_1 y} \quad (28)$$

$$e^{-\frac{c_2}{\lambda_D} \left(\frac{1}{T} - \frac{1}{\tau_{o2}} \right)} = 1 + \frac{r_2 - 1}{x - r_2 y} \quad (29)$$

$$e^{-\frac{c_2}{\lambda_D} \left(\frac{1}{T} - \frac{1}{\tau_{o3}} \right)} = 1 + \frac{r_3 - 1}{x - r_3 y} \quad (30)$$

Solving these equations we obtain the values of x , y and T . For simplicity the next symbols are introduced:

$$a = e^{\frac{c_2}{\lambda_D} \left(\frac{1}{\tau_{o1}} - \frac{1}{\tau_{o2}} \right)} \quad (31)$$

$$b = e^{\frac{c_2}{\lambda_D} \left(\frac{1}{\tau_{o1}} - \frac{1}{\tau_{o3}} \right)} \quad (32)$$

$$\frac{x}{y} = z \quad (33)$$

When equation 28 is divided by equation 29 and these symbols are introduced, we obtain:

$$y_1 = \frac{1}{1-a} \left\{ \frac{a(r_2-1)}{z-r_2} - \frac{r_1-1}{z-r_1} \right\} \quad (34)$$

In the same way from equations 28 and 30 we obtain:

$$y_2 = \frac{1}{1-b} \left\{ \frac{b(r_3-1)}{z-r_3} - \frac{r_1-1}{z-r_1} \right\} \quad (35)$$

Introducing several values for z and calculating values for y , we have two series of y_1 and y_2 . The subscripts 1 and 2 denote the values of y relating to r_1 , r_2 and r_1 , r_3 . From these relations, two y - z curves are obtained. The values of y and z are obtained at the intersection of the curves. And then, from equation 33 the value of x is obtained. Introducing these values to equation 26, the value of $F(r)$ is obtained. Introducing the value of $F(r)$ and τ_o into equation 22, we obtain the true temperature T of the flame.

At the light intensities of the flame, measured using a spectroscope, the following conditions are necessary.

(1) During the measurement, the values of x and y must not be changed. To meet this requirement during the measurement, the flame must be in a constant state, especially with respect to the density of the atoms of the resonance lines, the density of the solid or liquid particles, and consequently the thickness of the flame.

(2) The width of the slit that regulates the focus does not affect Kirchhoff's law, and it does not affect, in principle, the measurement of the flame temperature. However, the target position of the flame must be unchanged during the measurement.

Also the position of the spectroscope and the light path must be constant.

(3) With the above requirements it is difficult to strictly satisfy that of (1) because the burning state of the colored-flame compositions is not very stable. Therefore, to obtain the value of r with high accuracy, it is recommended to first obtain the values of r for several temperatures of the standard light and create an average line; second, select the values in the line as r_1 , r_2 , and r_3 .

Otherwise some special device is recommended to minimize the time of measurement. One can use a light wedge, setting it directly in front of the slit, although a photograph at the dry plate is useful for the analysis; however, in this case the surface of the light source must be perfectly flat, and the image of the flame must be large and of uniform brightness.

(4) The better the accuracy of the measured data, the better will be the accuracy of the calculated temperatures. Therefore, one must use a photometer as accurately as possible to measure the blackening density of the photographic dry plate when a photographic method is used. In such a case the characteristic curve of the plate must be also considered.

(5) To satisfy the conditions of equation 16, λ_n must be close to λ_D . Therefore, the dispersion of the spectroscope should be as large as possible to make operations of the measurement easy.

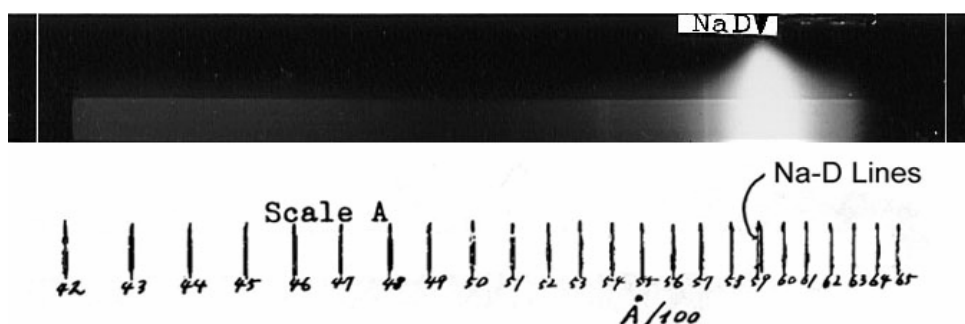


Photo 1. Continuous spectra appeared on both sides of the Na-D lines at lower temperatures.

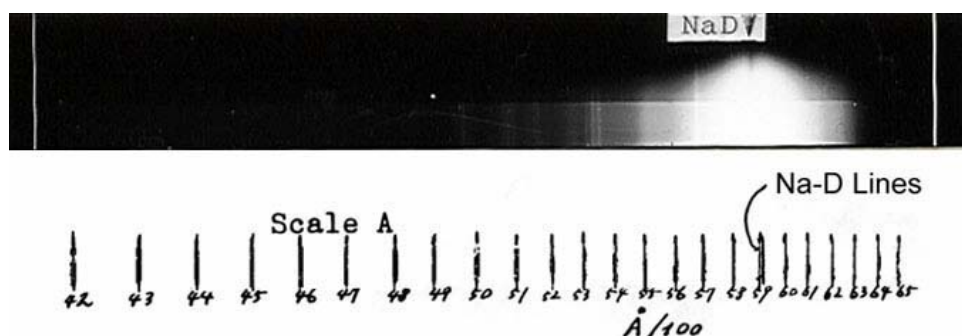


Photo 2. Continuous spectra appeared on both sides of the Na-D lines at higher temperatures.

(6) When the temperatures of the flame are expected to be close to those of the standard light, create a curve of τ_0 vs. r and extend it until it intersects the line of $r = 1$. The position of that intersection denotes the rough flame temperature. In this paper a calculated value is given for comparison (denoted with the symbol \odot mark in Figure 2).

(7) There is no need to add a Na source to the sample compositions, especially when flame temperatures exceed about 3000 K. Ordinary ingredients contain Na salts in small quantity as an impurity. Sodium plays a role in the resonance lines. At high temperatures, the Na source must be of very small quantity. If it is large, continuous spectra appear on both sides of the Na-D lines and interfere with measurement operations increasing errors. Photos 1 and 2 show sample results when a fairly large quantity of Na-salt is added.

The compositions used for the photos are:

For Photo 1	
Potassium perchlorate	75%
Colophony	15%
Sodium carbonate (anhydrous)	10%
For Photo 2	
Potassium nitrate	20%
Magnesium	60%
Sodium carbonate (anhydrous)	20%

Example 1. Potassium Perchlorate Composition

Composition: Potassium perchlorate (for industry use)	45%
Magnesium (smaller than 149 μ)	45%
Polyvinyl chloride	10%

Sample cartridge: The composition is loaded into a brown paper tube of 10 mm inside diameter, 96 mm long, having a wall thickness of about 0.5 mm.

Loading density : 1.21 g/cm³

Burning velocity: 3.58 mm/sec

Position of the temperature measurement:

In the middle at a point 10 mm above the flame base.

VI. Measurement and Examples of Calculation Using Method 2

In this paper the author used the photographic method although it is not as accurate. Using a spectroscope of 6.0 Å/mm dispersion at the Na-D lines and changing the temperature of a standard light in four stages, light beams were introduced into the spectroscope after passing through the flame, and the image was photographed. Afterward, a sensitization curve of the dry plate was prepared. On the dry plate the blackening densities of Na-D lines and near the lines were considered as energy values, and the ratio of the two was taken as the value of r . Details of the arrangements of instruments are described in the next part.

Black body temperatures of the standard light were adjusted to a : 2380 K; b : 2490 K, c : 2595 K and d : 2690 K. In each case light beams were introduced into the spectroscope through the flame and photographed on one dry plate as shown in Photo 3.

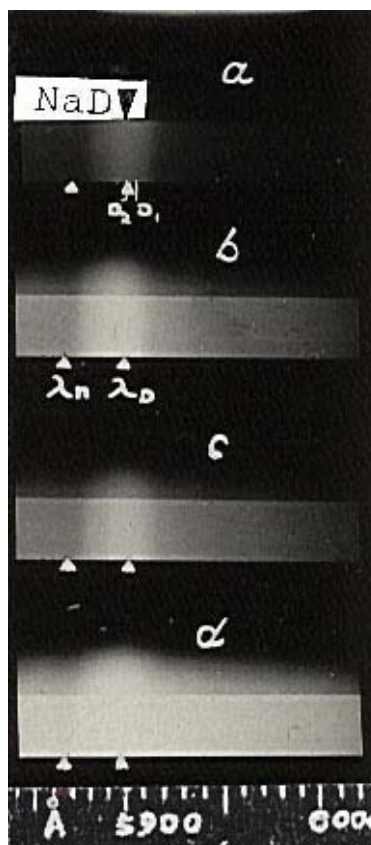


Photo 3. Reflections of the standard light when its temperature was changed in four stages through a sample flame.

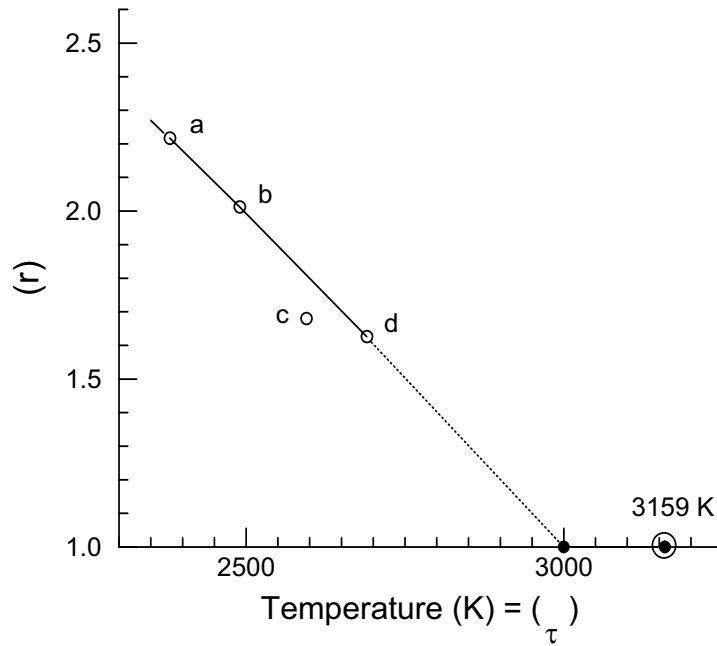


Figure 2. Graphical expression of the data measured.

Measurement:

In the photos, the Na-D₁ and Na-D₂ lines were fairly dispersed. This came from the adjustment of the spectroscope. The symbols denote the measured point of blackening at the wavelength λ_D or λ_n. Attention was paid to the measuring point so that the same points correspond in photos a, b, c, and d. The values thus obtained are shown in Figure 2.

In this case *c* denotes an irregular value and was omitted for calculation. The values of *r* which were useable are as follows:

d	τ _{o₁} = 2,690 K	r ₁ = 1.626
b	τ _{o₂} = 2,490 K	r ₂ = 2.012
a	τ _{o₃} = 2,380 K	r ₃ = 2.217

Calculations:

$$1/\tau_{o_1} = 0.0003717$$

$$1/\tau_{o_2} = 0.0004016$$

$$1/\tau_{o_3} = 0.0004202$$

$$c_2/\lambda_D = 1.4387/0.590 \times 10^{-4}$$

$$= 2.44 \times 10^4$$

$$a = e^{\frac{c_2}{\lambda_D} \left(\frac{1}{\tau_{o_1}} - \frac{1}{\tau_{o_2}} \right)} = e^{-0.7296} = 0.4821$$

$$b = e^{\frac{c_2}{\lambda_D} \left(\frac{1}{\tau_{o_1}} - \frac{1}{\tau_{o_3}} \right)} = e^{-1.1834} = 0.3062$$

$$y_1 = \frac{1}{1-a} \left\{ \frac{a(r_2-1)}{z-r_2} - \frac{r_1-1}{z-r_1} \right\}$$

$$= 1.931 \left\{ \frac{0.4879}{z-2.012} - \frac{0.626}{z-1.626} \right\}$$

$$y_2 = \frac{1}{1-b} \left\{ \frac{b(r_3-1)}{z-r_3} - \frac{r_1-1}{z-r_1} \right\}$$

$$= 1.441 \left\{ \frac{0.3726}{z-2.217} - \frac{0.626}{z-1.626} \right\}$$

As a test, several values of *z* are assumed, and when these are introduced to the above equations, we obtain:

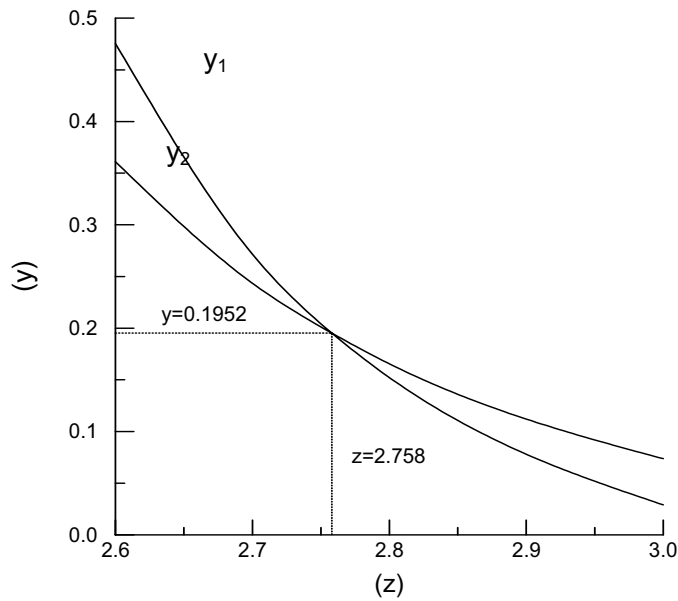


Figure 3. A graphical calculation of the y and z values.

z	y_1	y_2
2.60	0.3611	0.4755
2.70	0.2435	0.2716
2.80	0.1655	0.1522
2.90	0.1120	0.0780
3.00	0.0736	0.0290

From these values of y_1 and y_2 the graph shown in Figure 3 is obtained.

The point of intersection of the curves is at $z = 2.758$, $y = 0.1952$

From these values:

$$x = yz = 0.5384$$

From equation 26:

$$F(r) = 1 + \frac{r-1}{x-ry}$$

Introducing each value, we obtain:

$$F(r_1) = 3.833$$

$$F(r_2) = 7.946$$

$$F(r_3) = 12.503$$

And

$$2.303 \frac{\lambda_D}{c_2} = 2.303 \times \frac{0.590 \times 10^{-4}}{1.4387} = 0.9445 \times 10^{-4}$$

From equation 22:

$$T = \frac{1}{\frac{1}{\tau_o} - 2.303 \frac{\lambda_D}{c_2} \log F(r)}$$

Introducing values for $1/\tau_{o_1}$, $1/\tau_{o_2}$, $1/\tau_{o_3}$ and corresponding values for $F(r_1)$, $F(r_2)$, and $F(r_3)$, we have:

$$\text{from } \tau_{o_1}: T = 3,159 \text{ K}$$

$$\text{from } \tau_{o_2}: T = 3,159 \text{ K}$$

$$\text{from } \tau_{o_3}: T = 3,159 \text{ K}$$

The three data perfectly coincide with each other and show there is no error in the calculations. Namely, the temperature $T = 3,159 \text{ K}$ is the answer.

Example 2. Barium Nitrate Composition

Composition:	Barium nitrate (for industry use)	60%
	Magnesium (smaller than 149μ)	30%
	Shellac	10%

Sample cartridge: The same as in Example 1.

Loading density : 1.42 g/cm³

Burning velocity: 3.25 mm/sec

Position of the temperature measurement:
The same as in Example 1.

Measurement:

The relationship of τ_o to r from photographs is:

$$\tau_{o_1} = 2,690 \text{ K} \cdot r_1 = 1.264$$

$$\tau_{o_2} = 2,595 \text{ K} \cdot r_2 = 1.364$$

$$\tau_{o_3} = 2,380 \text{ K} \cdot r_2 = 1.669$$

Calculation:

$$1/\tau_{o_1} = 0.0003717$$

$$1/\tau_{o_2} = 0.0003854$$

$$1/\tau_{o_3} = 0.0004202$$

$$a = 0.7160, b = 0.3062$$

$$y_1 = 3.521 \left\{ \frac{0.261}{z - 1.364} - \frac{0.264}{z - 1.264} \right\}$$

$$y_2 = 1.441 \left\{ \frac{0.205}{z - 1.669} - \frac{0.264}{z - 1.264} \right\}$$

The relationship of z to y :

z	y_1	y_2
2.1	0.1373	0.2304
2.2	0.1063	0.1499
2.3	0.0842	0.1010
2.4	0.0690	0.0693
2.5	0.0567	0.0476
2.6	0.0472	0.0324

Making a graph of r vs. y , the values of the intersection point are:

$$z = 2.402, y = 0.0687$$

From these:

$$x = yz = 0.1650$$

Introducing the values of x , y and r into the equation for $F(r)$, we have:

$$F(r_1) = 4,376$$

$$F(r_2) = 6,105$$

$$F(r_3) = 14,300$$

Introducing these into the temperature equation, we have:

$$\text{from } \tau_{o_1} : T = 3,214 \text{ K}$$

$$\text{from } \tau_{o_2} : T = 3,213 \text{ K}$$

$$\text{from } \tau_{o_3} : T = 3,214 \text{ K}$$

The three values coincide with each other and show there is no error in the calculation. And $T = 3214 \text{ K}$ is the flame temperature for this example.

VII. Conclusion

The above descriptions are concluded as follows:

- (1) A general theoretical equation was developed that relates the ratios of the spectral light intensities of the resonance lines to that of neighboring positions under the assumption that Kirchhoff's law is applicable even when the flame contains small solid or liquid particles.
- (2) When the density of the solid or liquid particles is too large, the line-reversal method is not useful for measuring flame temperatures. However, in other cases, as long as the brightness of the standard light can be compared to that of the flame, the line-reversal method is useful for measuring flame temperatures. In this case the solid or liquid particles have no influence on the measurement.
- (3) Applying the general theoretical equation, the author planned to measure flame temperatures even when they were higher than those of the standard light.

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