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Ballistics of Firework Shells

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ABSTRACT

Heretofore we have had four important problems with calculations in this field, i.e., with the interior ballistics when using Black Powder as the propellant:

(1) to obtain a suitable form function of the propellant which consists of irregular grains,

(2) to obtain solutions when the burning rate of the propellant grains is proportional to P^{α}, where P is an internal pressure of the mortar barrel and α the pressure exponent,

(3) to obtain suitable solutions when the propellant gas escapes from the burning room through the clearance between the wall and the shell in the mortar, and with the exterior ballistics

(4) to obtain simply the drag coefficient for various shapes of shells.

For (1) a treatment to calculate the surface areas and volumes of grains assuming the propellant grains consist of a mixture of cubes and spheres is proposed. For (2) a method to solve a three order differential equation derived from three basic interior ballistic equations step by step with proper time intervals is proposed. For (3) the nozzle theory used for rocket engines is introduced. For (4) the fact that the maximum height of the projectile in the air is almost the same as that of vacuum when the flying times of the both are equal is applied.

These methods are applied to 6-inch shells and examined if they are suitable in practice.

Introduction

We cannot lift firework shells without the oldest Black Powder even today, because smokeless powder only burns in the mortar leaving the shell at the bottom unmoved. However, the development of theoretical treatment of the interior ballistics using Black Powder in this has been too slow. I tried in the past to solve this problem,[1] but it was only a shift because I used the method which had been used for smokeless powder to cannons.

The differences between the ballistic characters of black and smokeless powders may be in three points: a large burning rate of the former at the atmospheric pressure which is about ten times as large as that of smokeless powder, a low force of explosives of the former of about one third of the latter and a low pressure exponent value of about 0.5 which is about one half of the latter. I proposed here a step by step method to solve the ballistic equations with these points. In this process, the difference between the firework mortar and the cannon is also considered: the former has a rather large clearance between the wall and shell in the barrel which may cause an unnegligible gas flow out.

In the next, I proposed a method of exterior ballistics to find the drag coefficient of the shell having data of the muzzle velocity and the time of flight from the start to the fall on to the ground. This method may be sometimes useful.

Symbols for Interior Ballistics

A	"Vivacity" of propellant: the burning ratio dz/dt at one atmosphere
Ae	Clearance area between the wall and the shell in the barrel
C	Volume of the barrel
c	Volume of the barrel behind the shell at the time t
c_0	Initial volume of the barrel behind the shell at the time t = 0
d_b	Diameter of the barrel
d_s	Diameter of the shell
f	Force of explosives (Impetus)
g	Gravitational acceleration
i	Ballistic coefficient or denotes the end of the *i* th time interval
K	Adiabatic expansion constant
k	Ratio of the sectional area of the shell to that of the barrel
n	Average adiabatic expansion coefficient from 0 K to the gas temperature T K
P	Pressure in the barrel
P_0	Atmospheric pressure
V	Muzzle velocity of the shell
v	Velocity of the shell in the barrel
W	Weight of the shell
z	Burning ratio: ratio of the burnt mass to the initial mass of the propellant charge
x	Moving distance of the shell from the origin in the barrel
t	Time of movement of the shell
Δt	Small time interval for calculation
α	Pressure exponent coefficient
γ	Adiabatic expansion coefficient
δ	Density of the Black Powder grains
η	Co-volume
η_z	Practical co-volume
θ	Angle of inclination of the mortar from the horizontal
λ	Coefficient of the propellant mass in imaginary mass of the shell
μ	Imaginary mass of the shell including the propellant mass
Σ	Sum of values
σ	Sectional area of the barrel
$\varphi(z)$	Form function of the propellant grains
Ψ	Flow out coefficient of the gas
ω	Weight of the charge
$\dot{\omega}$	Weight of the flow out gas per second

Superscripts:

One dot (example: x'): Derivative with respect to time

Two dots (example: x''): Second derivative with respect to time

Three dots (example: x'''): Third derivative with respect to time

Subscripts or superscripts:

i End of the *i*-th time interval on calculation

i–1 Beginning of the *i*–1-th time interval on calculation

Form Function of Black Powder Grains

The theoretical form function of a propellant grain is generally expressed as $\varphi(z) = S/S_0$ as a function of z, where S is the surface area of a grain at a burning ratio z and S_0 that of the initial at $z = 0$. However, the grains of Black Powder are very irregular in shape and size and the ordinary method of calculation is not useful (Figure1).

A hypothesis is proposed that the powder is a mixture of cubic grains and spherical grains and the side length of the former is the same as the diameter of the latter (Figure 2).

Figure 1. Configurations of Black Powder grains between two sieve openings 0.850 mm and 1.000 mm. The scale is in mm. (Table 1. No. 8)

The burning times of the cube and sphere are the same at the same pressure, but the burning surface area of the former is about two times as large as that of the latter. Therefore, by mixing both of the grains in a proper ratio we have a hypothetical Black Powder which makes us easy to calculate the form function which could resemble that of the real Black Powder.

The real Black Powder is firstly divided into some ranks of size using sieves. The calculation for mixing the cubes and spheres is carried out as follows. For each group of size

$$n_c + n_s = N \qquad (1)$$

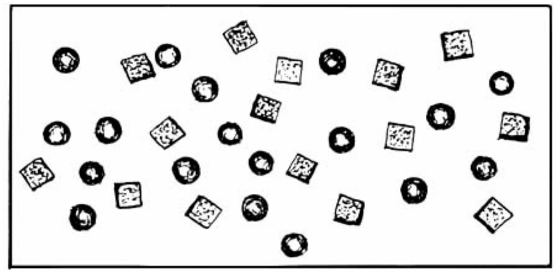

Figure 2. Hypothetical Black Powder grains which consist of cubes and spheres.

$$n_c w_c + n_s w_s = W_g \qquad (2)$$

where

- N is the total number of the grains in the real powder,
- n_c the number of cubes,
- n_s the number of spheres,
- W_g the total weight of the grains in the real powder,
- w_c the weight of a cube,
- w_s the weight of a sphere.

Solving these equations we have

$$n_s = \frac{N w_c - W_g}{w_c - w_s} \qquad (3)$$

$$n_c = N - n_s \qquad (4)$$

Table 1. Data of Grains of 1 kg Black Powder.

No.	Opening of sieve (mm)	Average size of a grain (mm)	Total weight of grains W_g (kg)	Number of grains N	Number of spheres n_s
1	0.175–0.210	0.193	0.000540	65,060	46,380
2	0.210–0.355	0.283	0.017555	452,448	20,687
3	0.355–0.425	0.390	0.089903	1,228,183	760,356
4	0.425–0.500	0.463	0.099759	813,032	501,033
S	0.500–0.600	0.550	0.190329	895,666	507,981
6	0.600–0.710	0.655	0.207554	666,519	513,002
7	0.710–0.850	0.780	0.196098	321,630	179,468
8	0.850–1.000	0.925	0.164907	177,186	122,006
9	1.000–1.180	1.090	0.030836	21,048	15,620
10	1.180–1.400	1.290	0.001444	651	560

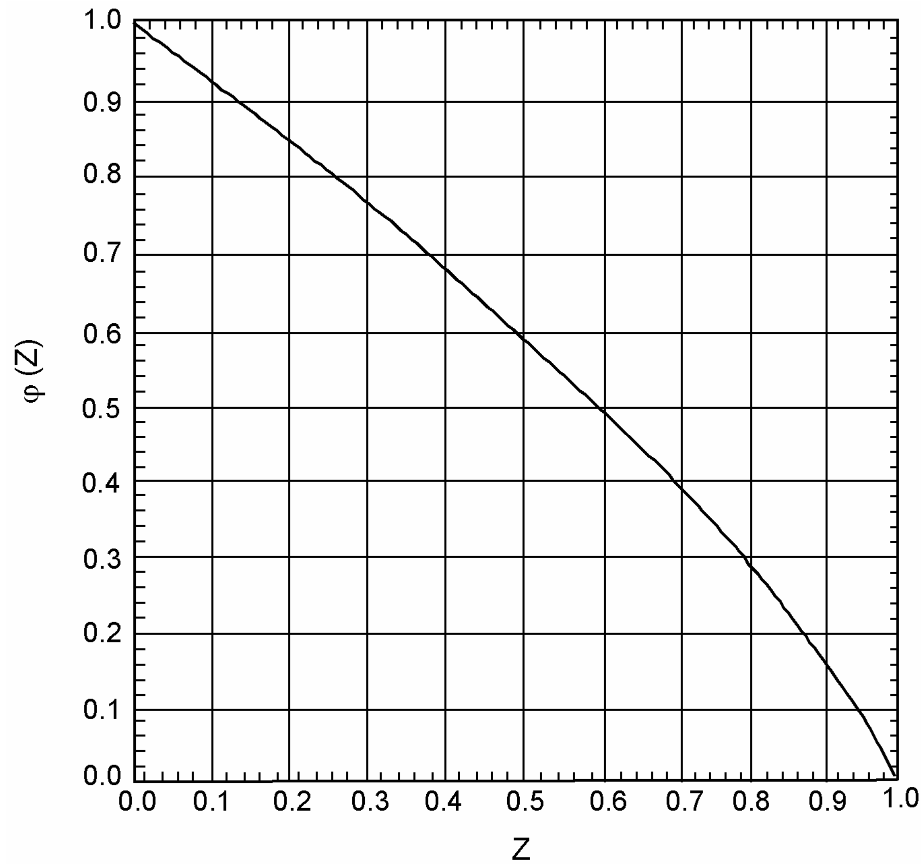

Figure 3. Form function of Black Powder grains due to a hypothetical mixture of cubes and spheres.

Example

Propellant: "Small Grain Black Powder" manufactured by Nippon Kayaku Co., which is most popularly used for lifting charge of firework shells in Japan.

The grains were ranked into 10 groups by sieving l00 grams of the powder using 11 sieves.

In Table 1 the average size of a grain was obtained by averaging the upper and lower openings of the sieves. When sieving, there was a small loss of powder, 1.3 grams, and the values of W_g were magnified proportionately with each obtained weight. A powdered part 0.001075 kg that passed the sieve of 0.175 mm is omitted. For each group the number of grains was practically counted with 1000 to 3000 and the total weight was measured. From these data the values of N were determined.

For the hypothetical mixture the weight of a grain was calculated by following formulas:

for the cube $w_c = 1.75 \times \ell^3$,

for the sphere $w_s = 1.75 \times 4/3\ \pi\ r^3$

where the value 1.75 is the density of the real Black Powder (g/cc), ℓ the side length of the cube, r the radius of the sphere where $2r=\ell$.

The burning velocity of the Black Powder was measured by burning three compressed blocks (density: 1.75 g/cm^3) of the sample powder in the open air and an average value of 9.52 mm/s was obtained. In the atmospheric pressure the grains will burn from their surface to each centre with this rate. The volume and the burning surface area were calculated with the time for each group. Multiplying these data by the number of grains n_c or n_s and summing these data with the time with each group and then with all groups, the sum of the volumes and the burning areas with all grains were ob-

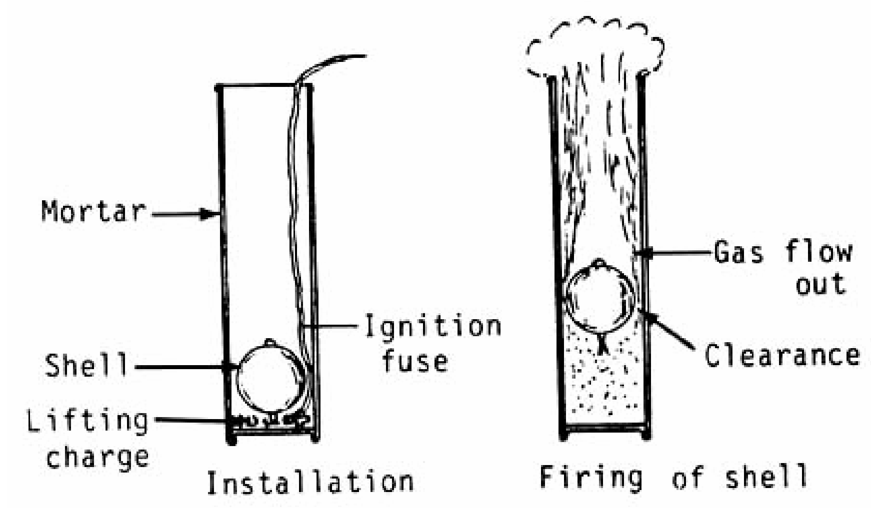

Figure 4. Loading and firing of shell.

tained as functions of time. The burning ratio z and the form function φ(z) were calculated by the following formulas:

$z = 1 - V/V_0$ for time t, (4)

$\varphi(z) = S/S_0$ for time t, (5)

where V is the sum of volumes, V_0 the initial value of V, S the sum of the burning surface areas, S_0 the initial value of S. In combination of (4) and (5) a curve of $z - \varphi(z)$ was obtained as it is shown in Figure 3.

Vivacity of Black Powder

Vivacity is a characteristic value which generally represents the burning ratio at one atmospheric pressure when the grains begin to burn. Using the data obtained by the studies of the form function, the value was determined as follows: [3]

$$\begin{aligned} A &= S_0\, w\, \delta \qquad (6) \\ &= 559.83 \times 0.0952 \times 1.75 \\ &= 93.27\ \mathrm{s}^{-1} \end{aligned}$$

where S_0 = 559.83 dm^2, w is the burning rate = 0.0952 dm/s, δ = 1.75 kg/dm^3.

Interior Ballistic Solution

The mortar is usually installed vertically on the ground as it is seen in Figure 4. In the installation the clearance around the shell is important to ignite the lifting charge from upside. When ignited, the gas and smoke firstly appears from the muzzle and then the shell.

The energy conservation is expressed as

$$\frac{f\left(\omega z - \int \dot{\omega}\, dt\right)}{n-1} = \frac{1}{2}\mu v^2 + \frac{\left(P - P_0\right)\left(c - \eta_z \omega\right)}{n-1} \qquad (7)$$

where

$$\eta_z = \frac{1}{\delta} + \left(\eta - \frac{1}{\delta}\right) z \qquad (8)$$

The equation of motion of shell is

$$\mu \frac{dv}{dt} = k\sigma\left(P - P_0\right) - W \sin\theta$$

$$\mu = i\frac{W}{g}\left(1 - \lambda\frac{\omega}{W}\right) \qquad (9)$$

Generally, the mortar is installed vertically on the ground and sinθ =1.

The burning of Black Powder is defined[2] as

$$\frac{\mathrm{d}z}{\mathrm{d}t} = A\varphi(z)\left(P/P_0\right)^{\alpha}, \quad \text{where } \alpha = 1/2 . \tag{10}$$

The gas flow out is expressed as[4]

$$\dot{\omega} = Ae\, P\Psi / f^{\frac{1}{2}}$$

$$\Psi = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \left\{2g\left(\frac{\gamma}{\gamma+1}\right)\right\}^{\frac{1}{2}} \tag{11}$$

under the condition

$$P_0 / P < \left\{2/(\gamma+1)\right\}^{\{\gamma/(\gamma-1)\}} \tag{12}$$

After the lifting charge has burnt out

$$P(c-\eta)^{\gamma} = K \tag{13}$$

By differentiating equation (7) with respect to t, we have

$$f\left(\omega\frac{dz}{dt} - \dot{\omega}\right) = (n-1)\ \mu x' \cdot x'' + \frac{dP}{dt}\left[\sigma x + c_0 - \left\{\frac{1}{\delta} + \left(\eta - \frac{1}{\delta}\right)z\right\}\omega\right] + (P - P_0)\left\{\sigma x'' - \left(\eta - \frac{1}{\delta}\right)\omega\frac{dz}{dt}\right\} \tag{14}$$

By differentiating equation (9)

$$\frac{dP}{dt} = \frac{\mu}{k\sigma} x''' \tag{15}$$

By substituting equation (15) for dP/dt in equation (14) we have equation 16:

From equation (9) the pressure is found:

$$P = \frac{\mu}{k\sigma} x'' + \frac{W}{k\sigma} + P_0 \tag{17}$$

The sum of the gas flow out could be expressed as

$$\int \dot{\omega}\ dt = \sum{}^{i}\ \frac{1}{2}\left(\dot{\omega}_{i-1} + \dot{\omega}_i\right)\ \Delta t \tag{18}$$

From equation (7) the burning ratio is found:

$$z = \frac{\int \dot{\omega}\ dt\ +\ \frac{n\text{-}1}{2}\mu x'^2 + (P - P_0)\left(\sigma x +\ c_0 - \frac{\omega}{\delta}\right)}{\omega\left\{f + \left(\eta - \frac{1}{\delta}\right)(P - P_0)\right\}} \tag{19}$$

It is possible to solve equation (16) in combination with equations (10), (11), (17), (18) and (19) by a step and step method. The calculation program was planned as it is shown in Table 2.

Table 2. A Program to Solve Interior Ballistic Equations for the Shell.

(1)	x'''_{i-1}
(2)	x''_{i-1}
(3)	x'_{i-1}
(4)	x_{i-1}
(5)	x'''_i
(6)	$\Delta x'' = ½\ \{(1) + (5)\}\ \Delta t$
(7)	$x''_i = (6) + (2)$
(8)	$\Delta x' = ½\ \{(7) + (2)\}\ \Delta t$
(9)	$x'_i = (8) + (3)$
(10)	$\Delta x = ½\ \{(9) + (3)\}\Delta t$
(11)	$x_i = (10) + (4)$

$$x''' = \frac{k}{\mu}\,\frac{f\left(\omega\frac{dz}{dt} - \dot{\omega}\right) - (n-1)\ \mu x' x'' - \sigma\ (P - P_0)\left\{x' - \left(\eta - \frac{1}{\delta}\right)\frac{\omega}{\sigma}\frac{dz}{dt}\right\}}{x + \frac{c_0}{\sigma} - \left\{\frac{1}{\delta} + \left(\eta - \frac{1}{\delta}\right)z\right\}\frac{\omega}{\sigma}} \tag{16}$$

(12)	$\frac{\mu}{k\sigma}\times\ (7)$
(13)	$(12) + W / k\sigma + P_0 = P_i$
(14)	$P_i - P_0 = (13) - P_0$
(15)	$\dot{\omega}_{i-1}$
(16)	$\sum^{i-1} \dot{\omega}\ \Delta t$
(17)	$P_0\ /P_i$
(18)	$\dot{\omega}_i = \left(Ae\ \Psi / f^{\frac{1}{2}} \right)\times\ (13)$
(19)	$\sum^{i} \dot{\omega}\ \Delta t = (16) + \frac{1}{2}\left\{(15)+(18)\right\}\Delta t$
(20)	$f \times \sum^{i} \dot{\omega}\Delta t = f \times (19)$
(21)	$\frac{n-1}{2}\mu v^2 = \frac{n-1}{2}\mu \times (9)^2$
(22)	$\sigma x = \sigma \times (11)$
(23)	$c = c_0 + \sigma x = c_0 + (22)$
(24)	$c - \frac{\omega}{\delta}$
(25)	$(P_i - P_0)\left(c - \frac{\omega}{\delta} \right) = (14) \times (24)$
(26)	(20) + (21) + (25)
(27)	$\left(\eta - \frac{1}{\delta}\right)\left(P_i - P_0\right) = \left(\eta - \frac{1}{\delta}\right)\times(14)$
(28)	$(27) + f$
(29)	$\omega \times (28)$
(30)	$z = (26) / (29)$
(31)	$\varphi\ (z)$: from Figure
(32)	$\left(\frac{P_i}{P_0}\right)^{\alpha} = \left(\frac{(13)}{P_0}\right)^{\frac{1}{2}}$
(33)	$\frac{\mathrm{d}z}{\mathrm{d}t} = A\ \varphi(z) \times \left(\frac{P_i}{P_0}\right)^{\frac{1}{2}}$ $= A \times (31) \times (32)$
(34)	$\omega_\mathrm{i}\frac{\mathrm{d}z}{\mathrm{d}t} = \omega_\mathrm{i} \times\ (33)$

(35)	$\omega_\mathrm{i}\frac{\mathrm{d}z}{\mathrm{d}t} - \omega_\mathrm{i} =\ (34) - (18)$
(36)	$f\left(\omega_i\frac{\mathrm{d}z}{\mathrm{d}t} - \dot{\omega}\right) = f \times (35)$
(37)	$(n-1)\mu\cdot x'\cdot x'' = (n-1) \times (9) \times (7)$
(38)	$\left(\eta - \frac{1}{\delta}\right)\frac{\omega}{\sigma}\frac{\mathrm{d}z}{\mathrm{d}t} = \left(\eta - \frac{1}{\delta}\right)\frac{\omega}{\sigma}\times(33)$
(39)	$x - \left(\eta - \frac{1}{\delta}\right)\frac{\omega}{\sigma}\frac{\mathrm{d}z}{\mathrm{d}t} = (9) - (38)$
(40)	$\sigma\left(P_i - P_0\right)\left\{x' - \left(\eta - \frac{1}{\delta}\right)\frac{\omega}{\sigma}\frac{\mathrm{d}z}{\mathrm{d}t}\right\}$ $= \sigma \times (14) \times (39)$
(41)	(36) – (37) – (40)
(42)	$\left(\eta - \frac{1}{\delta}\right)z = \left(\eta - \frac{1}{\delta}\right)\times(30)$
(43)	$\left(\eta - \frac{1}{\delta}\right)\ z\ + \frac{1}{\delta} = (42) + \frac{1}{\delta}$
(44)	$\left\{\left(\eta - \frac{1}{\delta}\right)\ z\ + \frac{1}{\delta}\right\}\frac{\omega}{\sigma} = (43) + \frac{\omega}{\sigma}$
(45)	$x + \frac{c_0}{\sigma} - \left\{\left(\eta - \frac{1}{\delta}\right)\ z\ + \frac{1}{\delta}\right\}\frac{\omega}{\sigma}$ $= (11) + \frac{c_0}{\sigma} - (44)$
(46)	$x''' = \frac{k}{\mu}\times (41) / (45)$

In the program, the values of (1)–(4), (15), and (16) come from the former interval. Firstly, a value of x_i''' which is expected from the values of the former intervals, is put into (5). However, at the first interval we cannot expect the value. Therefore we use a proper value of x_i''' in this case, for example x_i''' = 500,000,000 dm/s^3.

With a time interval Δt (Example: 0.0001 s) the calculation proceeds from (1) to (46). The value of (46) is introduced again to (1) and the calculation is repeated. On repeated iterations, the value of x_i''' approaches a definite value or the difference of the value from the former becomes allowable. Then we proceeded to the next time interval.

Example

Data: A 6 inch round shell, weight: 1.25 kg, inside length of mortar: 103cm

(1)	f	=	0.2934×10^{6} dm·kg/kg[2]
(2)	A	=	$93.268\ \mathrm{s}^{-1}$
(3)	$\varphi(z)$		from Figure 5
(4)	ω	=	0.0750 kg
(5)	W	=	1.250 kg
(6)	g	=	$98.0\ \mathrm{dm/s^2}$
(7)	i	=	1
(8)	μ	=	$i\dfrac{W}{g}\left(1+\lambda\dfrac{\omega}{W}\right)=0.01314\dfrac{\mathrm{kg\cdot sec^2}}{\mathrm{dm}}$
(9)	σ	=	$\pi d_b^2/4 = 1.887\ \mathrm{dm}^2$
(10)	s	=	$\pi d_s^2/4 = 1.584\ \mathrm{dm}^2$
(11)	Ae	=	$\sigma - s = 0.303\ \mathrm{dm}^2$
(12)	s/σ	=	k = 0.8394
(13)	P_0	=	$103.33\ \mathrm{kg/dm^2}$
(14)	δ	=	$1.75\ \mathrm{kg/dm^3}$
(15)	η	=	$0.983\ \mathrm{dm^3/kg}$
(16)	γ	=	1.214
(17)	n	=	1.260

The results are summarized in Figure 5.

A Method for the Calculation of the Drag Coefficient of a Shell in Air

Having the data of muzzle velocity and flying time from the start to the fall on to the ground, the drag coefficient of the shell is simply calculated. Generally the movement of a projectile in the air is expressed by equations (20) and (21). When the shell is vertically fired on the ground, sinθ = 1, and (21) could be omitted.

$$\frac{W}{g}\frac{\mathrm{d}v}{\mathrm{d}t} = -K\ s\ v^2 - W\ \sin\theta \qquad (20)$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -\frac{g\ \cos\theta}{v} \qquad (21)$$

K is defined with the drag coefficient C_D as

$$K = C_D \times \frac{1}{2}\frac{\rho}{g} \qquad (22)$$

From (20), formulas (23), (24) and (25) are derived.[5]

$$a = (W/KS)^{½} \qquad (23)$$

$$x = V/a \qquad (24)$$

$$H = \frac{a^2}{2g} \times 2.303\ \log(x^2+1) \qquad (25)$$

Due to the fact that the maximum height is almost the same as that in the vacuum with the same flying time from the start to the fall [6]

$$H_{(Vacuum)} = \frac{1}{8} g\ T_{(Air)}^2 = H \qquad (26)$$

In the equations

θ	inclination from the horizontal,
K	a constant which includes the drag coefficient C_D,
C_D	the drag coefficient,
ρ	density of air
V	the muzzle velocity of the shell,
a	a parameter,
x	a parameter,
H	the maximum height,
$H_{(Vacuum)}$	the maximum height in the vacuum,
$T_{(Air)}$	the flying time in the air,
S	the sectional area of the shell.

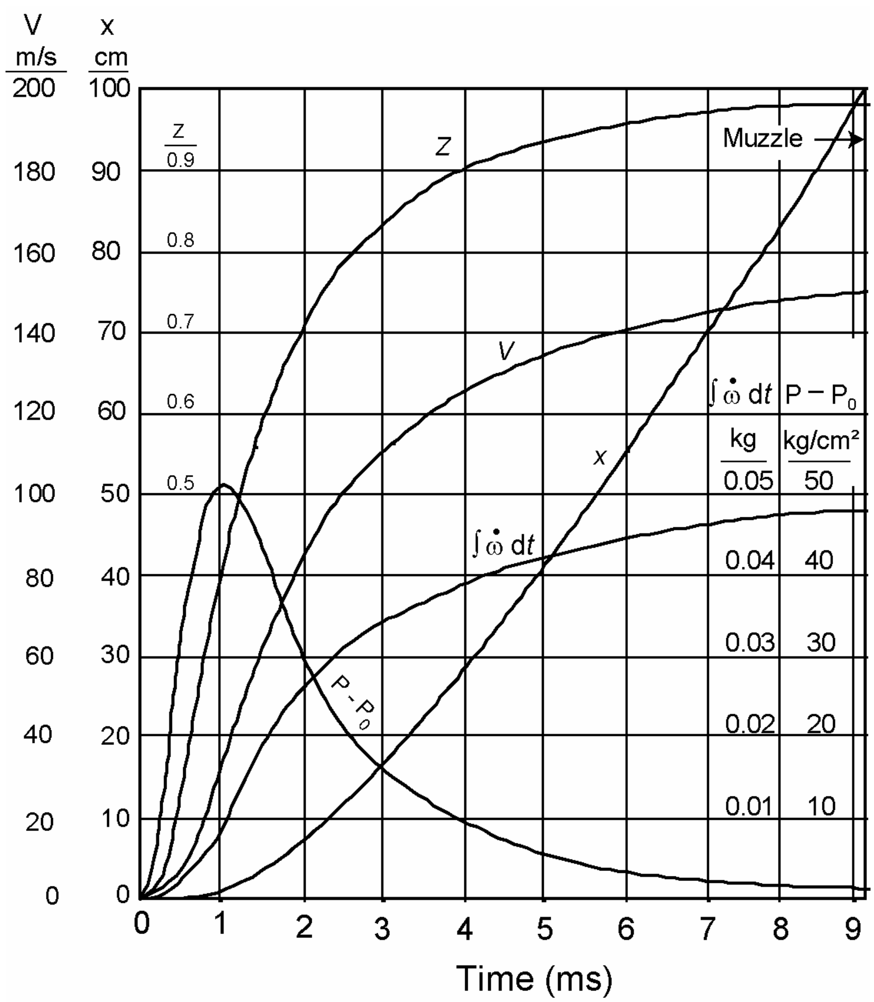

Figure 5. Results of calculation for a 6-inch shell.

When the velocity in the air does not exceed 250 m/s, it could be assumed that the value of C_D is a constant.

By substituting (24) for a in (25) we have

$$\frac{H}{V^2} = 0.1175 \frac{1}{x^2} \log(1+x) \tag{27}$$

For the equation (27) a diagram is prepared as Figure 6.

Example

Data: A 6-inch round shell,

S	$0.01584\ m^2$
g	$9.80\ m/s^2$
ρ	$l.280\ kg/m^3$
V	$129\ m/s$
$T_{(Air)}$	$14.5\ s$
W	$1.250\ kg$

From (26)

$H = 9.80 \times 14.5^2 / 8 = 258,$

therefore

$H/V^2 = 258/129^2 = 0.0155.$

From Figure 6 $x = 2.55$ is obtained.

From (24)

$a = 129/2.55 = 50.59$.

From (23)

$K = 1.250/(50.59^2 \times 0.01584) = 0.03084$.

From (22)

$C_D = 0.03084 \times 2 \times 9.8 / 1.280 = 0.472$.

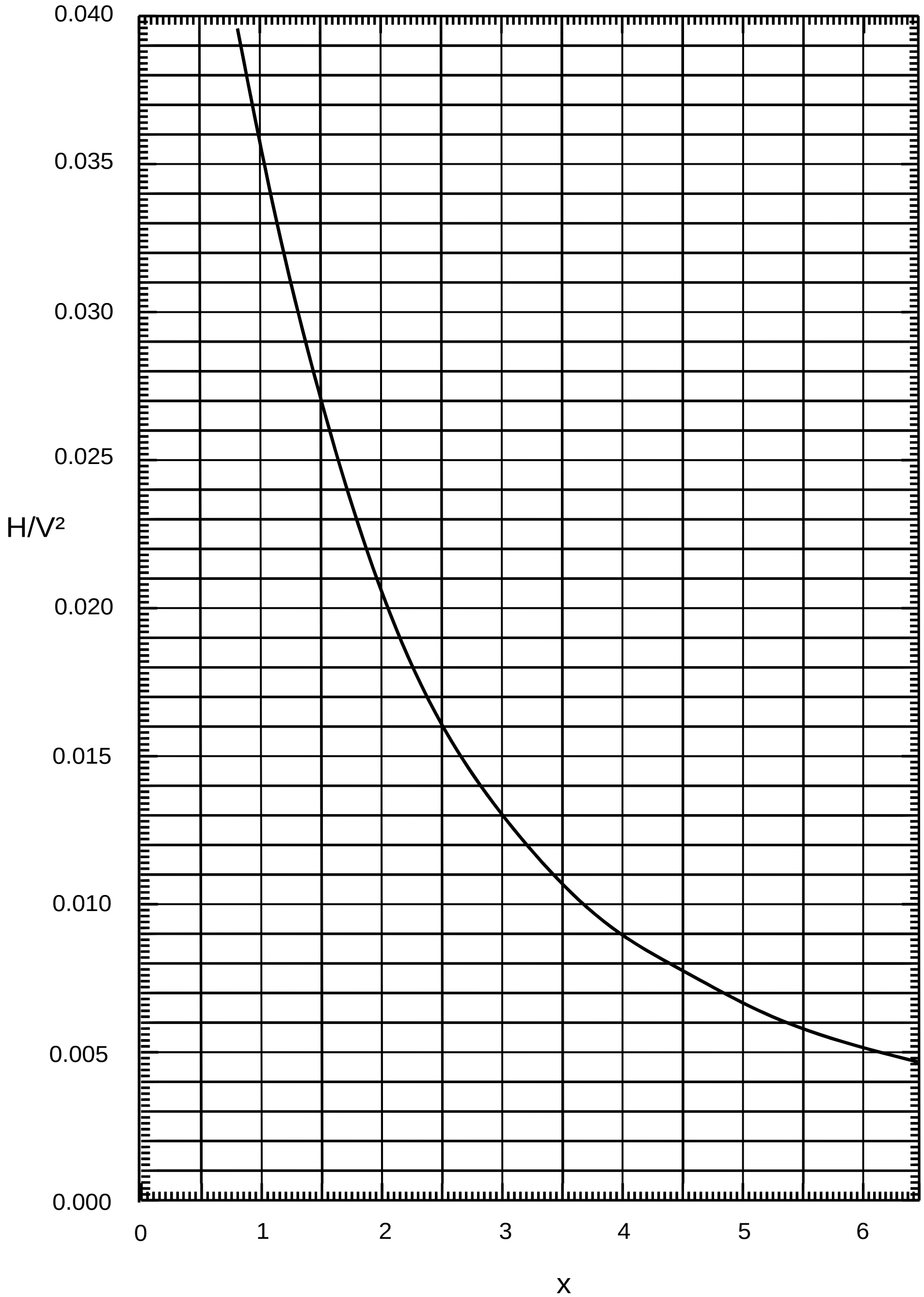

Figure 6. Diagram for equation (27).

Discussion and Conclusion

A method to determine the form function for irregularly shaped Black Powder grains by using a concept of a hypothetical mixture which consists of cubes and spheres is proposed. It may be a way to treat an irregularly shaped material by calculation. There is no way to prove if the mixture exactly resembles the real Black Powder in the surface estimation, but this method seems useful.

A step by step method for solving interior ballistic equations under the condition the pressure exponent $\alpha = 1/2$ is proposed. Comparing the results of calculation for a 6-inch shell with those of my past experiment, it is found that the muzzle velocity from this calculation is 150 m/s when the weight of the shell is 1.25 kg, while those of the experiment were 120 m/s when the weight was 1.25 kg and 152 m/s when the weight was 0.61 kg.[7] Therefore, when we use this method of calculation, the value of the ballistic coefficient should be 2.0 (equation (9))

In the calculation, the gas flow out has been considered, however, the escape of the grains has been ignored. The reason why the value of the ballistic coefficient is so large may be in that in the case of firing, a little quantity of the grains escape out of the barrel without effect. The quantity of the escaped grain is estimated by a calculation for the example as about 4.6 grams.

A method of calculation of the drag coefficient by a simple process having the data of the muzzle velocity and the flying time from the start to the fall on to the ground with shells. The value of the drag coefficient of a 6-inch shell calculated as an example is 0.472 and it is almost the same as those of my past experiments.[8] The muzzle velocity must be known before the calculation: it may be a handicap of this method, however, it is sometimes useful when we hope to obtain simply the coefficient for various shapes of shells. When we measure the muzzle velocity of a shell, the instruments must be carefully installed not being disturbed by the gas from the muzzle, because the gas with smoke flows out faster than the shell.

The reason why only Black Powder is used for lifting firework shells is in that it burns very fast even at the atmospheric pressure due to the large value of the vivacity A and raises the pressure in the loading room very rapid, although there is a fairly large clearance between the wall and shell in the barrel. The value of the vivacity of Black Powder is about 200 times as large as that of smokeless powder.

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