# Peak In-Mortar Aerial Shell Accelerations 

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#### Abstract

Internal mortar pressures were measured for a range of somewhat typical fireworks aerial shell firing conditions. These data were used to determine the peak shell accelerations produced during firing. Under the conditions investigated, peak aerial shell acceleration ranged from approximately 4 to $16 \mathrm{~km} / \mathrm{s}^{2}$ (400 to 1600 times the acceleration due to gravity) and appeared to be mostly independent of nominal shell size.


As a check on the acceleration results, the same mortar pressure data were used to calculate aerial shell muzzle velocities. These were found to be in close agreement with measured velocities.

Keywords: fireworks, aerial shell, acceleration, muzzle velocity, mortar pressure, pressure impulse

## Introduction

For safety reasons, a fireworks display operator needs to know that aerial shells leave the mortar at high speed. Further, it is important to know approximately how far the aerial shells can travel. However, it is not important for the operator to know the rate of acceleration of aerial shells within mortars as they are fired. Similarly, except to know that the acceleration is great and the resulting inertial forces on the shells are large, the shell manufacturer does not need detailed knowledge of the magnitude of aerial shell acceleration. Nonetheless, it is sometimes a topic of discussion, and knowledge of these accelerations would satisfy the curiosity of a number of individuals. This short article is intended to help satisfy that curiosity.

Several years ago data was collected, albeit for another purpose, that can be used to calculate the acceleration of aerial shells while being fired from mortars. These data are internal mortar pressures as a function of time for various shell parameters (e.g., size and shape, lift type and mass, and shell mass). At the same time, the muzzle velocity of the shells was measured and can be used as a check on the calculated shell accelerations. Some examples of the basic data and the results produced are presented in this article.

## Background

If the forces acting on a body are known, it is a simple matter to calculate the acceleration produced. Pressure has the units of force per area; for example, Newtons per square meter (also termed Pascals and abbreviated Pa). Accordingly, the force $(F)$ acting on an aerial shell with a known cross-sectional area ( $A$ ) perpendicular to the pressure gradient, when experiencing a pressure difference $(P)$ between one side and the other is ${ }^{[1]}$

$$
\begin{equation*}
F=P \cdot A \tag{1}
\end{equation*}
$$

Then simply by rearranging Newton's second law of motion, and knowing the mass ( $m$ ) of the aerial shell, the acceleration (a) it experiences can be calculated as

$$
\begin{array}{ll}
a=\frac{F}{m} & \text { or by substitution } \\
a=\frac{P \cdot A}{m} & \tag{3}
\end{array}
$$

Figure 1 is an example of the pressure measured inside a mortar as a shell is being fired. Because the pressure is not constant during the firing, neither is the acceleration of the shell. Nonetheless, equation 3 accurately predicts the


Figure 1. Typical internal mortar pressure during the firing of an aerial shell.
acceleration at every instant, providing the mortar pressure at the same instant is used. Thus, the shell's acceleration reaches a maximum when the mortar pressure peaks, and this peak acceleration can be calculated using equation 3 .

In the same tests where mortar pressures were measured, aerial shell muzzle velocities were also measured. This provided an opportunity to indirectly confirm the accuracy of the peak acceleration determinations by using the mortar pressure data also to predict the measured muzzle velocities.

In general, for any body, its change in velocity $(v)$ in response to a time dependent acceleration can be represented by

$$
\begin{equation*}
v_{f}-v_{i}=\int_{t_{i}}^{t_{f}} a(t) \mathrm{dt} \tag{4}
\end{equation*}
$$

where the subscripts $i$ and $f$ are for initial and final values. For an aerial shell initially at rest (stationary), substituting for acceleration using equation 3 , and integrating over the time of exposure to the pressure in the mortar equation 4 becomes

$$
\begin{equation*}
v_{m}=\frac{A}{m} \int_{t_{i}}^{t_{e}} P(t) \mathrm{dt} \tag{5}
\end{equation*}
$$

where $v_{f}$ is now muzzle velocity $\left(v_{m}\right)$ and $t_{f}$ is now the time of exiting $\left(t_{e}\right)$, see Figure 1.

The integral in equation 5 is usually referred to as pressure impulse $\left(I_{p}\right)$. In these tests, values for the pressure impulse were determined and
used to calculate the aerial shell muzzle velocities from equation 6 .

$$
\begin{equation*}
v_{m}=\frac{A}{m} I_{p} \tag{6}
\end{equation*}
$$

## Experimental

For uniformity, all of the test shells for this project were assembled using molded plastic shell casings. Nominal shell size ranged from 3 to 8 inches. Most shells were spherical in shape, but some 3- and 4 -inch shells were cylindrical. In an attempt to have the spherical shells perform in a similar manner to typical oriental shells, the lift powder used was a fairly homogeneous blend of powder harvested from a collection of shells manufactured in China. The lift powder for the cylindrical shells was 2FA fireworks Black Powder manufactured by Goex. ${ }^{[2]}$ The air temperature at the time of firing ranged from 21 to $27^{\circ} \mathrm{C}$ ( 70 to $80^{\circ} \mathrm{F}$ ). The tests were conducted at about $1400 \mathrm{~m}(4600 \mathrm{ft})$ above sea level, resulting in air pressure of approximately 850 mbar. Additional mortar and shell test information is provided in Table 1.

All mortars were steel with piezoelectric pressure gauges installed in the mortar plug. In this way the internal mortar pressures were measured as the shells were fired. ${ }^{[3]}$ The mortars were also fitted with a series of trip wire sensors to detect the passage of the shell after exiting the mortar. Signals from the trip wires controlled a series of time counters to produce the data used to calculate velocities of the shells as they exited the mortar. ${ }^{[4]}$

The test results are reported in Table 2. In each case, the peak mortar pressure reported was the highest value from the digital pressure data. Pressure impulse is the integral of the pressure data, starting from the first sign of pressure rise $\left(t_{i}\right)$ and ending at the point of shell exit $\left(t_{e}\right)$ (such as identified in Figure 1). The measured velocity of the exiting shell was determined by noting the time taken for the shell to travel a known distance after exiting the mortar. The calculated shell velocity was determined by substituting the measured pressure impulse and the known cross-sectional area and mass for the aerial shell into equation 6 . The peak shell ac-

Table 1. General Test Shell and Mortar Information.

| Nominal Shell Size (in.) | Mortar Diameter $(\mathrm{mm})$ | Mortar Length (m) | Shell Shape | Shell <br> Diameter <br> $(\mathrm{mm})$ | Shell Mass (g) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 79 | 0.51 | Spher. | 66 | 135 |
|  |  |  | Cylin. | 67 | 180 |
| 4 | 103 | 0.61 | Spher. | 95 | 350 |
|  |  |  | Cylin. | 92 | 500 |
| 5 | 129 | 0.76 | Spher. | 119 | 620 |
| 6 | 154 | 0.76 | Spher. | 144 | 1140 |
| 8 | 203 | 0.91 | Spher. | 193 | 2700 |

To convert inches to millimeters, multiply by 25.4 .
To convert millimeters to inches, divide by 25.4.
To convert meters to inches, multiply by 39.4.
To convert grams to pounds, divide by 454.
celeration was determined from equation 3 , using the measured peak mortar pressure.

To be consistent with the general reliability of the data, in Table 2 peak pressures were reported to the nearest 10 kPa , pressure impulses were reported to the nearest $0.1 \mathrm{kPa} \cdot \mathrm{s}$, measured and calculated muzzle velocities were reported to the nearest $5 \mathrm{~m} / \mathrm{s}$, and peak accelerations were reported to the nearest $1 \mathrm{~km} / \mathrm{s}^{2}$.

## Discussion

The aerial shells had been assembled such that their mass, the type and amount of lift powder, and the mortar specifications were fairly representative of typical aerial shells. However, caution is warranted in applying the results of these tests in situations where any of the conditions are different.

An examination of the results for the series of 4-inch cylindrical shells provides an indication of the general reliability of these data. Note that

Table 2. Test Shell Firing Results.

| Nominal Shell Size (in.) | Shell Shape | Lift Mass (g) | Peak Pressure (kPa) | Pressure Impulse (kPa.s) | Measured Velocity ( $\mathrm{m} / \mathrm{s}$ ) | $\begin{gathered} \hline \hline \text { Calculated } \\ \text { Velocity } \\ (\mathrm{m} / \mathrm{s}) \\ \hline \hline \end{gathered}$ | Peak Acceleration (km/s ${ }^{2}$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | Spher. | 28 | 430 | 4.5 | 80 | 85 | 8 |
|  | Cylin. | 28 | 500 | 5.0 | 90 | 95 | 10 |
| 4 |  | 28 | 210 | 3.2 | 65 | 65 | 4 |
|  | Spher. | 46 | 660 | 5.9 | 125 | 125 | 14 |
|  | Cylin. |  | 880 | 7.9 | 110 | 105 | 12 |
|  |  | 50 | 1200 | 8.0 | 110 | 105 | 16 |
|  |  |  | 970 | 7.9 | 100 | 105 | 13 |
|  |  |  | 770 | 7.5 | 100 | 100 | 10 |
| 5 | Spher. | 50 | 610 | 5.9 | 100 | 105 | 11 |
| 6 | Spher. | 85 | 680 | 7.4 | 110 | 110 | 10 |
| 8 | Spher. | 155 | 830 | 11.2 | 120 | 125 | 9 |

To convert inches to millimeters, multiply by 25.4 .
To convert grams to ounces, divide by 28.3 .
To convert kilopascals to pounds per square inch, divide by 6.89 .
To convert meters per second to feet per second, multiply by 3.28 .
while the peak pressures (and peak accelerations) for these firings varied considerably, the pressure impulses (and thus muzzle velocities) were in relatively close agreement. The authors have seen this same type of large variability in peak mortar pressure, yet reasonably consistent overall performance, in numerous other con-fined-combustion measurements. The reason for this effect is not clear but is suspected to be the result of small dynamic differences in the ignition and initial flame spread within the pyrotechnic charge (an interesting subject, but beyond the scope of this article.)

There was relatively close agreement between measured and calculated shell muzzle velocities, not only for the 4 -inch cylindrical shells, but all others as well. Further, the muzzle velocities were reasonably close to $100 \mathrm{~m} / \mathrm{s}(330 \mathrm{ft} / \mathrm{s})$, regardless of shell size. This is consistent with the results reported by Shimizu, ${ }^{[5]}$ Contestabile, ${ }^{[6,7]}$ and in unpublished results of the authors. Thus there is a reasonably high degree of confidence in the reported results.

The maximum shell accelerations typically ranged from 8 to $12 \mathrm{~km} / \mathrm{s}^{2}$ and appear to be mostly independent of nominal shell size. In part, the $4 \mathrm{~km} / \mathrm{s}^{2}$ value reported in Table 2 was a result of using a smaller than normal amount of shell lift powder. However, it may also be a reflection of the widely varying peak pressures thought to result from the differences in ignition and flame spread mentioned above. Similarly, the 14 and $16 \mathrm{~km} / \mathrm{s}^{2}$ values may again be the result of these same differences.

These peak acceleration results can be put into perspective, recalling that the acceleration due to gravity is $9.8 \mathrm{~m} / \mathrm{s}^{2}$. Accordingly, at their maximum acceleration, these somewhat typical aerial shells were experiencing approximately 400 to 1600 times the acceleration due to gravity. Obviously, this produces powerful forces on the contents of the shell (so-called set-back forces) that communicate to the shell's casing as well. For example, in the relatively new "Lampare" style aerial salutes (maroons), a container is generally filled with liquid fuel, combined in some fashion with a charge of flash powder. Consider a liquid fuel with a density of $0.85 \mathrm{~g} / \mathrm{cc}$ that is placed in a container with a height of 150 mm (about 6 inches). If that shell is propelled such
that it receives the peak acceleration seen in the tests reported above, the liquid pressure at the bottom of the container would range from 0.5 to 2.0 MPa ( 70 to 290 psi ). Thus it is clear why some fuel containers fail catastrophically during such shell firings and why the fuel containers typically are strongly encased.

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## Notes and References

1) This is a simplification in that things such as drag forces produced by the combustion gases rushing past the exterior of the shell are not considered. However, this is not expected to introduce a significant error in the results being reported.
2) Goex, Inc., PO Box 659, Doyline, LA 71023, USA.
3) The pressure gauges were PCB Piezotronics 101A04, connected to PCB Piezotronics 480D06 amplifying power supplies. The data were recorded using Fluke Scopemeters model 95 or 97 , and then transferred to a computer for processing.
4) K. L. and B. J. Kosanke, "Measurement of Aerial Shell Velocity", American Fireworks News, No. 157 (1994). Also in Selected Pyrotechnic Publications of K. L. and B. J. Kosanke, Part 3 (1993 and 1994), Journal of Pyrotechnics, 1996.
5) T. Shimizu, Fireworks, From a Physical Standpoint, reprinted by Pyrotechnica Publications (1985) p 183.
6) E. Contestabile, "A Study of the Firing Characteristics of High Altitude Display Firework Shells", CANMET, MRL 88-020 (OPJ) (1988).
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